Do Only Fundamentals Matter for Stock Prices? – A Case Study for the AH Premium in the Chinese Stock Markets

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Abstract

The high and volatile stock price differences between the dual-listed AH-shares of the connected Shanghai-Hong Kong markets since November 2014, namely AH premium, represent a challenge to the viewpoint that only fundamentals matter for stock prices. We first show the difficulties of fitting the data by various present-value asset pricing models. To reconcile with the data, we then propose an internal rationality learning model, in which agents don’t know the price mapping and optimally have different subjective beliefs about the future capital gains in the two markets. Convergence traders are highly likely to suffer a loss in this situation.

Key Words: AH Premium, Shanghai-Hong Kong Stock Connect, Present-value Asset Pricing, Internal Rationality Learning

JEL: G12, G15
1. Introduction

This paper studies the AH premium, which is the price differences of dual-listed stocks in the Chinese stock markets; it is an interesting phenomena appropriate for analyzing whether only realized or also perceived fundamentals matter for stock prices. The stocks listed in the mainland China stock exchanges (Shanghai and Shenzhen) are called A-shares, and the ones listed in the Hong Kong exchange are called H-shares. Currently, there are 101 companies dual-listed in both A-share and H-share markets, most of which are sizable, especially the state-owned enterprises, accounting for 20% of the total market value in the A-share market. The shares of these dual-listed companies are called AH-shares. More importantly, the fact that AH-share stocks are identical with respect to shareholder rights, such as voting and profit-sharing, helps us control the fundamentals and focus on other potential explanations for the price differences in AH-shares, namely the AH premium. The Hang Seng China AH Premium Index plotted in Figure 1 measures the weighted average price ratio of AH-shares. The index of 100 means that A-shares are trading at par with H-shares, while an index larger than 100 indicates A-shares traded at a premium in comparison to H-shares, and one smaller than 100 indicates A-shares traded at a discount in comparison to H-shares. Figure 1 shows that AH-share prices are always different despite having the same fundamentals in both A-share and H-share markets.

Before November 2014, the Shanghai and Hong Kong markets were segmented in the sense that mainland investors were not allowed to invest in the Hong Kong market and foreign investors were barred from investing in the Shanghai market. The price differences of dual-listed stocks in the segmented markets have been extensively studied in the literature. Fernald and Rogers (2002) attribute the price discount seen in Chinese B-share stocks (i.e.,
only for foreigners) relative to A-share stocks (i.e., only for the citizens) to the fact that Chinese investors have a higher discount factor than foreigners. Chan, Menkveld and Yang (2008) offer the evidence that the AB-share premium is caused by foreign investors, who trade B-shares, have an informational disadvantage relative to domestic investors, who trade A-shares. At the same time, Mei, Scheinkman and Xiong (2009) propose that trading due to investors’ speculative motives can help explain a significant fraction of the price differences between the perfectly segmented dual-listed AB-shares. Most recently, Jia, Wang and Xiong (2017) empirically show that investor reactions to analysts’ recommendations are affected by their social connections, and that the investors’ differential reactions produce the AH premium in segmented markets.

The previously segmented Shanghai and Hong Kong markets, however, were connected through the initiation of the Shanghai-Hong Kong Stock Connect program in November 2014. Mainland investors can participate in the Hong Kong market through this connect program, as can Hong Kong and international investors in the Shanghai market. The connect program together with the dual-listed AH-share stocks provide us a natural experiment investigating whether only fundamentals matter for stock prices. If the answer is yes, the AH premium index should have converged to 100 according to the standard asset pricing theory. However, it actually diverged dramatically to almost 150 at peak and then fluctuated between 120 and 150. With regard to the cross-sectional individual stocks, 97 individual stocks out of 101 have larger premium for A-share prices after the connection than before. What is equally interesting is that A-share and H-share prices retain almost the same shape as Figure 2. During the price divergence period both A-share and H-share prices increased, but the quicker increase of the former relative to the latter contributed to the divergence. The same pattern was observed in the declining speed during the convergence period after July 2015.

The price differences of dual-listed stocks in connected markets not only exists in the Chinese stock markets, but also in other countries. For example Froot and Dabora (1999) show the price differences of three twin stocks dual-listed in both the US and the European
stock markets. In this paper, we focus on the AH-shares because they have more than 100
dual-listed stocks, a much larger sample than other stock markets.

This paper first investigates whether the present-value asset pricing models with agents’
heterogeneities and financial market frictions can quantitatively generate a sufficiently high,
volatile and persistent AH premium. Heterogeneities across agents could be reflected in
agents’ different discount factors, risk aversions (Fernald and Rogers 2002), and diverse
beliefs (Chan, Menkveld and Yang 2008; Chan, Menkveld and Yang 2009; Jia, Wang and
Xiong 2017), while financial market frictions across markets could represent transactions
cost (Froot and Dabora 1999) and dividend taxes. In all of these models, agents know the
mapping from the fundamentals to stock prices. Hence regardless of the heterogeneities and
financial market frictions, stock prices always equal to the discounted sum of expected future
fundamentals. In these present-value models, we find the following: (1) the above-mentioned
heterogeneities across agents cannot produce any price difference; (2) in addition to the
proposed heterogeneities, the generalized model shows that no price difference will occur
for any kind of heterogeneity across agents; (3) financial frictions across markets are able
to generate an almost constant 6% premium, which is however quantitatively not enough.
The AH premium in connected markets can be interpreted as a challenge to the present-
value models where only realized or perceived fundamentals matter for the prices (realized
fundamental models e.g. Campbell and Cochrane 1999 and Bansal and Yaron 2004; perceived
fundamental models e.g. Scheinkman and Xiong 2003 and Barberis et al. 2015).

The failure of present-value models to produce AH premium motivates us to propose
an internal rationality learning model based on Adam, Marcet and Nicolini (2016), in which
present-value formulas do not hold. Agents do not know the mapping from the fundamentals
to stock prices and optimize their behaviors based on their subjective beliefs about variables
that are beyond their control. Given these subjective beliefs, agents behave as speculators
and optimally update their expectations about capital gains using the Kalman filter. In this
situation, agents can have different beliefs about capital gains across A-share and H-share
stocks, as generated by differently perceived signal-noise ratios of stock price components in
the two markets. In turn, agents’ subjective expectations influence equilibrium stock prices,
and the realized stock prices feed back into agents’ beliefs. When agents have different
subjective beliefs, we aim to show that the self-referential property of the presented learning
model can generate data-like price differences.

Finally, we investigate the convergence traders’ strategy, which wagers on the price
convergence of similar or identical assets. A typical convergence trader will short sell an AH-
share in the Shanghai market, and long buy it in the Hong Kong market when it is worth
more in Shanghai. However, our learning model aims to show that convergence traders have
a large probability of suffering loss, since stock prices cannot converge in the short-run.

This paper contributes to the literature as being the first one to propose a structural
model to quantitatively study the dual-listed stock price differences in connected markets.
The mechanism does not rely on any Chinese institutional characteristics; it can thus be used
to study similar phenomena in other countries’ stock markets such as the so-called "Siamese
twin" stocks described by Froot and Dabora (1999). Furthermore, we also contribute to
the internal rationality learning literature by providing a scenario to support the internal
rationality learning mechanism for asset pricing (e.g. Adam, Marcet and Nicolini 2016 and
Adam, Marcet and Beutel 2017). And we introduce cross-learning scheme enriching belief
updating rules. Lastly, we contribute to the complete market literature by developing a
simulation-based computational method for trading positions in state-contingent bonds with
and without diverse beliefs.

The paper is structured as follows. Section 2 gives a brief introduction into the Chinese
stock markets. Section 3 argues that the present-value asset pricing models cannot generate
any price difference without financial market frictions. Section 4 shows that the prominent
financial market frictions existing in the real world are far from satisfactory when it comes
to producing high, volatile and persistent price differences. Section 5 uses an internal rational-
ity learning model to explain these price differences. Section 6 shows the implications for
Figure 1: Hang Seng China AH Premium Index

Figure 2: Hang Seng China AH-A and AH-H Price Index
convergence traders who may incur a big loss. Section 7 concludes this paper.

2. Overview of the Chinese Stock Markets

The Chinese mainland stock market is relatively young and started off in 1990 with the establishment of two stock exchanges: the Shanghai Stock Exchange (SSE) and the Shenzhen Stock Exchange (SZSE). The number of listed companies was only 13 initially. During the period from 1990 to 2015, the Chinese economic growth averaged an annual 10% GDP growth and undoubtedly led to the rapid development of financial markets. The market value of the Chinese stock market (excluding Hong Kong and Taiwan) reached $8.4 trillion at the end of 2015 and became the second largest one in the world, even though the ratio of market capitalization to GDP remained relatively low at about 60%. The number of listed companies also rose to 2,827. The main boards of the Shanghai and Shenzhen Stock Exchanges now list larger and more mature stocks, similar to the New York Stock Exchange (NYSE) in the US. The SZSE also includes two other boards, the Small and Medium Enterprise Board and the ChiNext Board, also known as the Growth Enterprise Board, which provides capital for smaller and high-technology stocks, and is similar to the NASDAQ in the US. Before the initiation of Shanghai-Hong Kong Stock Connect Program, mainland investors could only invest in A-share stocks, while Hong Kong and international investors could only invest in B-share stocks in Chinese mainland stock market.

Figure 3 shows the changes in the stock price index in mainland Shanghai and Shenzhen markets from 1995 to 2015. Mainland stock prices experienced two episodes of obvious boom and bust, one in 2006-2007 and the other in 2014-2015. The stock price index reached its historical peak in 2007 having risen from its bottom in 2005, and then quickly busting. Then, from 2009 to 2014 the market generally trended down. Allen et al. (2015) argue that the performance of the Chinese stock market has been disappointing, especially compared with the rapid GDP growth of the country. The market price boomed again in the second half of
2014, and almost doubled by mid-2015. One distinguishable characteristic of China’s stock market is that stock trading is new to most of participants, 80% of whom are individual investors (Mei, Scheinkman and Xiong 2009). Given the typical Chinese investor’s lack of experience, it is reasonable to hypothesize that these investors would often not be aware of the stock pricing mapping and hence would behave more like speculators. The larger volatility of the Chinese stock market relative to the US one is presented in Table 1 and supports this hypothesis.

3. Present-Value Models with Heterogenous Agents

In this section, we build a series of heterogeneous present-value models without financial market frictions. Across agents, we consider heterogeneities in the discount factors, risk aversions, and beliefs related to the fundamentals. We will show that it is difficult for heterogeneous present-value models, in which agents know the price mapping, to produce a high, volatile, and persistent AH premium in the connected markets.
3.1 Models in the Complete Market

We present a Lucas “tree” model with two types of agents, where the “tree” pays dividends. We first consider a complete market economy because it is simple and will illustrate the main point clearly. With the aid of analytical solutions, we will show that heterogeneities among the agents cannot contribute to any price difference.

3.1.1 Rational Expectation

We assume there exist two types of agents in the economy without loss of generalization. Type \( i \) agents in the economy account for a fraction of \( \mu^i > 0 \) of population \( i \in \{1, 2\} \) respectively, where \( \mu^1 + \mu^2 = 1 \). Type 1 agents stand for mainland investors and type 2 agents for Hong Kong and international investors. The two types may differ with respect to their degree of risk aversions, discount factors. We assume that the agents within each group are homogeneous.

Investors’ portfolios include A-shares, H-shares, and state-contingent bonds. Agents trade A-share and H-share stocks with each other. \( S^{1,A}_t, S^{1,H}_t, S^{2,A}_t, S^{2,H}_t \) represent A-and H-share stocks that type 1 and type 2 agents buy in period \( t \). One unit of an A-share and a H-share has the same fundamentals and pays the same dividend to investors as denoted by:

\[
D^A_t = D^H_t = D_t.
\]

For illustration purposes and without loss of generalization, we assume that the exogenous dividend process in the complete market economy is \( i.i.d \) taking the two values of high dividend \( D_h \) and low dividend \( D_l \) in each period, where \( \text{Prob}(D_l) = \pi, \text{Prob}(D_h) = 1 - \pi \). Agents have rational expectation and are well-informed about the dividend processes and the economy structure. Arrow securities \( B_t(D_h) \) and \( B_t(D_l) \) complete the market. During the period \( t + 1 \), \( B_t(D_h) \) pay 1 unit of consumption if the dividend payment is high; similarly
for $B_t(D_l)$. The Arrow securities market-clearing conditions are:

$$
\mu^1 B_t^1(D_j) + \mu^2 B_t^2(D_j) = 0 \forall j = h, l
$$

Both agents can participate in each asset’s market. The supply of each share is assumed to be 1 so that the A- and H-share market clearing can be denoted by:

$$
\mu^1 S_t^{1,Z} + \mu^2 S_t^{2,Z} = 1 \forall Z = A, H.
$$

For each period, the weighted average of consumption for the two agents is equal to the total dividend; hence the commodity goods market-clearing condition is:

$$
2D_t = \mu^1 C_t^1 + \mu^2 C_t^2.
$$

The utility function satisfies the standard property and is increasing, concave, and continuously differentiable. Type $i$ agents maximize their lifetime utility, which is subjected to the budget constraint

$$
\max_{\{C_t, S_t^{i,A}, S_t^{i,H}, B\}} E_0 \sum_{t=0}^{\infty} (\delta^i)^t u_i(C_t^i)
$$

s.t. 

$$
S_t^{i,A} P_t^A + S_t^{i,H} P_t^H + C_t^i + B_t^i(D_h)Q_t(D_h) + B_t^i(D_l)Q_t(D_l)
$$

$$
= S_{t-1}^{i,A} (P_t^A + D_t^A) + S_{t-1}^{i,H} (P_t^H + D_t^H) + B_{t-1}^i
$$

Hence, type $i$ agents consume $C_t^i$ amount of goods, buy $S_t^{i,A}$ units of A-shares, $S_t^{i,H}$ units of H-shares and Arrow securities with price $Q_t(\cdot)$ during each period, and receive income from the shares and Arrow securities bought in the previous period. The first-order conditions
with respect to Arrow securities lead to the standard full-insurance condition:

$$\delta^1 \frac{u^1_c(C^1_{t+1})}{u^1_c(C^1_t)} = \delta^2 \frac{u^2_c(C^2_{t+1})}{u^2_c(C^2_t)},$$

(1)

where $u^i_c$ is the marginal utility for the type $i$ agents.

The full-insurance condition highlights a complete market model. Although agents could have different discount factors $\delta^i$ and risk aversions $u^i$, the condition of full insurance implies a unique stochastic discount factor ($SDF$). Hence we can use either type agent’s $SDF$ to price the two shares. The agents have rational expectations; therefore they know the mapping from the fundamentals to prices, which represents the present-value formula of the stock prices. In this situation, the same dividends of the two shares along with the unique $SDF$ generate no price difference between the A-share and the H-share. Therefore, heterogeneous discount factors and relative risk aversions across two agents are not able to bring about any price difference between the connected markets.

Hereinafter, we develop a simulation-based computational method different from Judd, Kubler and Schmedders (2003) to show how the full insurance is achieved through two agents’ state-contingent-bond trading. For illustration we perform an exercise in which both agents have CRRA utility with the same discount factor while type 1 agents are more risk-averse than type 2 agents.\(^1\)

The full insurance is obtained by type 1 agents buying low state-contingent bonds and selling high state-contingent bonds, while type 2 agents are doing the opposite. Intuitively type 1 agents want a smooth consumption more than type 2 agents because type 1 agents are more risk-averse. We confirm this by calculating the quantity of state-contingent bond holdings, and find that type 1 agents’ consumption are relatively smoother across states than type 2 agents’. These observations are shown in Table 2. The algorithm is presented in more detail in Appendix A.1.1.

\(^1\)With state-contingent bonds, shareholdings of A-shares and H-shares are indeterminate because the shares are considered as ‘redundant’ assets. Therefore, we can keep agents’ shareholdings of the two assets fixed over time.
<table>
<thead>
<tr>
<th></th>
<th>$C(D_h)$</th>
<th>$C(D_l)$</th>
<th>$B(D_h)$</th>
<th>$B(D_l)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1 Agents</td>
<td>0.8736</td>
<td>0.5514</td>
<td>-0.0036</td>
<td>0.0988</td>
</tr>
<tr>
<td>Type 2 Agents</td>
<td>1.1264</td>
<td>0.4486</td>
<td>0.0036</td>
<td>-0.0988</td>
</tr>
</tbody>
</table>

Table 2: Consumption and State-Contingent-Bond Holdings in Two States

3.1.2 Diverse Beliefs in Fundamentals

There is a popular narrative in the market that foreign investors are pessimistic about the Chinese economy, while mainland Chinese citizens have more optimistic views. This is probably because mainland Chinese investors have an informational advantage relative to international investors, and are more aware of the actual performance of the Chinese economy. Stock brokers and market analysts tend to propagate this kind of story as a way to rationalize the AH premium existence.

In this subsection we analyze the price difference implications when two agents have diverse beliefs in fundamentals and are not informed about the objective probability as in the previous subsection. To this end, we depart from the full information rational expectation model and adopt a model in which two agents are endowed with diverse beliefs in dividends. However, despite this disagreement regarding dividend payments, they agree on the price mapping from dividends to the stock prices.

Let us assume type 1 agents are more optimistic about the fundamentals, which is consistent with the market narrative. Formally, type 1 agents perceive $\text{Prob}(D_l) = u$, while type 2 agents perceive $\text{Prob}(D_l) = v$ where $u < v$. More importantly, let us assume type 1 agents are right relative to type 2 agents, i.e. $(\frac{v}{u})^\pi (\frac{1-u}{1-v})^{1-\pi} < 1$. We will show that type 1 agents will take advantage of their information superiority so that they can accumulate assets and consume more goods. Following this, the first-order conditions with respect to the Arrow security lead to

$$u^1_c(C^1_{t+1}) = \left[\frac{v}{u}1(D^h_{t+1}) + \frac{1-v}{1-u}1(D^l_{t+1})\right] \delta^2 u^1_c(C^1_t) \delta^1 u^2_c(C^2_t)$$

where $[\frac{v}{u}1(D^h_{t+1}) + \frac{1-v}{1-u}1(D^l_{t+1})]$ is denoted by $A_{t+1}$ for simplicity; $1(D_h)$ and $1(D_l)$ are the
indicator functions which take the value 1 when $D_h$ and $D_l$ happen, respectively. The assumption that type 1 agents are relatively right implies that the consumption ratio of type 1 agents relative to type 2 agents is increasing over time, and in the long run type 2 agents will be driven out of the market. Rearranging (2) leads to

$$\delta^1 \frac{u^1_c(C^1_{t+1})}{u^1_c(C^1_t)} = A_{t+1} \delta^2 \frac{u^2_c(C^2_{t+1})}{u^2_c(C^2_t)}$$

which links the SDFs of two type agents. It implies that there is one unique subjective SDF in the complete market; hence both type agents are marginal investors. Again we could use either agent’s SDF to price any asset. Denote price of A-shares discounted by type 1(2) agents’ SDF by $P^{1,A}_t(P^{2,A}_t)$, we show $P^{1,A}_t = P^{2,A}_t$ as is in Appendix A.2; similarly, for H-shares. Due to the fact that A-shares and H-shares deliver the same amount of dividends in each period and there exists a unique SDF in the complete market, A-shares and H-shares should have the same value. Hence, the diverse beliefs about dividends cannot lead to any price difference.

We also use our computational method as a way of simulating the model and finding out how agents can get full insurance through state-contingent bonds trading. The algorithm used by Judd, Kubler and Schmedders (2003) is not appropriate for the diverse beliefs models. The algorithm in this case is the one detailed in Appendix A.1.2. Type 1 agents are more accurate with respect to the distance from true probability, hence they accumulate assets and consume more while type 2 agents accumulate debt and consume less. In the long run, type 1 agents consume the total dividends while type 2 agents get nothing. This is consistent with Bloom and Easly (2006). We find that it is the relative correctness of the perceived beliefs that drives this bond-trading pattern rather than their degree of optimism. The transition path for the state-contingent-bonds holdings is shown in Figure 4. However, even if the two agents have diverse beliefs about the economic fundamentals, the price difference would be 0.
3.2 A General Model

We have not addressed all possible heterogeneities and we have only considered the complete market environment so far because it is impossible to examine every alternative. This section lays out a general model to show that it is impossible to obtain a price difference in any frictionless present-value model.

We propose that heterogeneities across agents without financial frictions cannot generate any price difference. This proposition implies that the prices of the two shares with the same dividend streams are identical in each period when there are no financial market frictions (such as various transaction costs), even though heterogeneities across agents exist. This proposition holds for both a complete market and an incomplete market.

The proof for this proposition is as follows: Suppose type \( i \) agents are the marginal investor in the A-share market; then due to first-order conditions, we have

\[
P_t^A = E_t^i f(SDF_t^i, P_{t+1}^A, D_{t+1})
\]

where \( f \) is a generic function serving just for easy explanation. \( E_t^i \) can capture any possible expectation of type \( i \) agents related to the fundamentals, including the expectations detailed...
by Scheinkman and Xiong (2003) and Barberis et al. (2015). $SDF_i$ can be of any type, including the well-known habit utility $SDF$ described by Campbell and Cochrane (1999) or the Epstein-Zin utility $SDF$ used in Bansal and Yaron (2004). The market environment can be either complete or incomplete.

The above first-order condition also applies to H-shares, otherwise type $i$ agents would experience arbitrage opportunity. If type $i$ agents are the marginal investor in the A-share market, they will price H-share as well. This leads to the equation:

$$P^H_t = E^i_t f(SDF^i_t, P^H_{t+1}, D_{t+1}) .$$

We use $m_t$ to denote the marginal investor who prices both assets in period $t$ in equilibrium:

$$m_t = \arg \max_{i \in \{1, 2\}} E^i_t f(SDF^i_t, P_{t+1}, D_{t+1}).$$

The mapping from the fundamentals to stock prices, which is known by the agents, is obtained as follows:

$$P^A_t = E_t^{m_t} g((SDF^m_{t+j-1})^\infty_{j=1}, \{D_{t+j}\}^\infty_{j=1})$$

$$P^H_t = E_t^{m_t} g((SDF^m_{t+j-1})^\infty_{j=1}, \{D_{t+j}\}^\infty_{j=1})$$

where $g$ represents the corresponding abstract present-value mapping.

When the only existing heterogeneities are across agents, prices for A-shares and H-shares are identical. Intuitively, if there are no differences across the two shares (or markets), then they are the same goods in the perspective of investors who know the price mapping. In that case, regardless of how equilibrium prices are determined in the present-value models, no price difference should exist. Diverse beliefs, different discount factors, and different risk aversions among the two type agents will not create a price difference. We will consider the financial market frictions as market differences in Section 4.
4. Financial Market Frictions

The real world is not free of the financial frictions of the market. Among the frictions we will consider further on, the most prominent one is the dividend tax. According to the existing regulations, mainland investors pay a constant 5% and 20% dividend tax for A-shares and H-shares, respectively. Hong Kong and other international investors face a 10% dividend tax for both A-shares and H-shares. We use $\tau^{1,A}$, $\tau^{1,H}$, $\tau^{2,A}$, $\tau^{2,H}$ to represent the dividend taxes. The agent offering a higher price will be the marginal agent as a result of no short-selling constraints. Due to the dividend tax scheme, the after-tax dividend payment of A-shares is higher for type 1 agents than for type 2 agents. Hence, type 1 agents are marginal for A-shares and type 2 agents are marginal for H-shares, as can also be seen in the first-order conditions below. Without loss of generalization, we assume that all agents have log utility. A complete market is the first case we analyze in the situation which includes a dividend tax.

With the addition of a dividend tax, the budget constraints of a complete market model become:

$$S_t^{i,A} P_t^A + S_t^{i,H} P_t^H + C_t^i + B_t^i(D_h)Q_t(D_h) + B_t^i(D_t)Q_t(D_t)$$

$$= S_{t-1}^{i,A}(P_t^A + (1 - \tau^{1,A})D_t^A) + S_{t-1}^{i,H}(P_t^H + (1 - \tau^{i,H})D_t^H) + B_{t-1}^i, \forall i$$

For each period, the Euler equations in this case with no short-selling constraints become:

$$P_t^A = E_t^{i_1} \frac{C_t}{C_{t+1}} [P_{t+1}^A + (1 - \tau^{1,A})D_{t+1}] \quad S_t^{1,A} \geq 0$$

$$P_t^A > E_t^{i_2} \frac{C_t}{C_{t+1}} [P_{t+1}^A + (1 - \tau^{2,A})D_{t+1}] \quad S_t^{2,A} = 0.$$
Similarly, for H-shares the equations are:

\[ P_t^H > E_t \delta^1 \frac{C_t^1}{C_{t+1}^1} [P_{t+1}^H + (1 - \tau^{1,H})D_{t+1}^H] \quad S_t^{1,H} = 0 \]

\[ P_t^H = E_t \delta^2 \frac{C_t^2}{C_{t+1}^2} [P_{t+1}^H + (1 - \tau^{2,H})D_{t+1}^H] \quad S_t^{2,H} \geq 0 \]

Hence, the A-share price is the present value of the future dividend discounted by the SDF of the type 1 agents, while the price of H-shares is discounted by the SDF of the type 2 agents, since type 1(2) agents are the marginal person for the A-share (H-share) every time. Hence we obtain the present-value formula:

\[ P_t^A = E_t \sum_{j=1}^{\infty} (\delta^1)^j \prod_{k=1}^{j} \frac{C_t^{1+k-1}}{C_t^1} (1 - \tau^{1,A}) D_{t+j} \] (3)

\[ P_t^H = E_t \sum_{j=1}^{\infty} (\delta^2)^j \prod_{k=1}^{j} \frac{C_t^{2+k-1}}{C_t^2} (1 - \tau^{2,A}) D_{t+j} \] (4)

In a complete market environment, the SDFs of the two agents have an identical value during each period. The dividend tax is constant and can be factored out, thus leading to:

\[ \frac{P_t^A}{P_t^H} = \frac{1 - \tau^{1,A}}{1 - \tau^{2,H}} \]

The price ratio is constant over time with an approximate value of 105.6%, which are inconsistent with the observation that the AH premium index fluctuates between 100% and 150%. Furthermore it is worthwhile to notice that, if \( \tau^{1,A} = \tau^{1,H} \) and \( \tau^{2,A} = \tau^{2,H} \), then the prices would be identical. Hence, if both mainland and international investors were subject to the same dividend taxes for both A- and H-shares, then there would be no price difference in this complete market framework, even if dividend taxes were to differ across agents.

Typically we cannot obtain analytical solutions for the price differences under incomplete market conditions, because we do not have the equation that links the two agents’ SDFs.
After including the dividend tax, the budget constraints in the incomplete market become:

\[ S^i_A P^A_t + S^i_H P^H_t + C^i_t = S^i_{t-1} (P^A_t + (1 - \tau^i_A) D^A_t) + S^i_{t-1} (P^H_t + (1 - \tau^i_H) D^H_t) \] \( \forall i \)

When the two types of agents have the same level of risk aversion and the same discount factors, type 1 agents will be the marginal person for A-shares, while type 2 agents will be the marginal for H-shares. In this case, we obtain the same results as for the complete market model with a constant price ratio of 105.6%. Even with allowing for different discount factors and levels of risk aversion across the two agents, the maximal price ratio which could be reached is 118.75%, which is the ratio of post-tax dividends for type 1 agents, when type 1 agents are the marginal investor in both markets. This is still not able to match the peak 150% AH premium index observed in the data.

In addition to the dividend tax, we also take into account the transaction cost. Generally, the transaction cost includes the financial tax, the cost of changing currency, and the expected changes in the exchange rate. We have several observations. First, the total financial tax in the Hong Kong stock market is about 0.118%, while the one in Shanghai is about 0.169%. Second, the currency change cost is less than 0.5% due to the Shanghai-Hong Kong Connect Program. Finally, the Hong Kong Dollar is expected to appreciate, rather than depreciate, against the RMB by an average of 1.64%, as measured by the exchange rate future during the period of interest. In summary, such a small transaction cost cannot produce desirable quantitative results.

The government intervention is regarded as another argument for the price differences. Some economists and market participants hold the long-standing view that the Chinese central government directly and frequently intervenes in the mainland stock market. However, this is not true. Since 2005, it has only happened once, namely when the A-share stock market bubble burst at the end of June 2015. The Chinese government required state-owned
investment banks to support stock prices by adopting long positions, as a way to avoid a severe financial crisis and due to concerns about the high leverage held by many Chinese investors. When stock prices stabilized in August, the Chinese government’s direct intervention promptly ended. The government intervention in the Hong Kong stock market during the 1998 Asian financial crisis is also well-documented.

Another suggested argument is that of liquidity, where the higher is a stock’s liquidity, the higher is its price. One popular measure of liquidity is the proportion of no-price-change days for a stock over a certain period (Mei, Scheinkman and Xiong 2009). Based on daily data for the period 2006-2016, the proportion of trading days with no price changes for A-shares was 0.65%, while the corresponding proportion for H-shares averaged 1.05%. This suggests that A-shares are just marginally more liquid than H-shares. We doubt that the small difference in liquidity can quantitatively produce such a high and volatile AH premium. Furthermore, the difference in market liquidity is perhaps endogenous, having been caused by different investors’ subjective beliefs about future capital gains, as described by Adam et al. (2015). In Section 5, we will present a similar model to show that different investors’ subjective beliefs in future capital gains could be the explanation for the AH premium. The correlation between the AH premium and the stock liquidity could be a result of both factors being influenced by the same subjective beliefs.

Throughout the rest of the paper, we will provide a model beyond the present-value one, in order to better understand the AH premium. When agents do not have the perfect knowledge about the price mapping from the fundamentals to stock prices, then they behave like speculators and can have different subjective beliefs when it comes to the capital gains between the A-share and H-share markets; these beliefs often match the way bankers, traders, and other Chinese citizens view the stock markets.
5. An Internal Rationality Learning Model

Sections 3 and 4 have shown that present-value asset pricing models, where agents know the price mapping, are not able to generate a sufficiently high, volatile and persistent AH premium. This section presents an "internal rationality" learning model where agents do not know the price mapping.

5.1 Model Environment

The environment is as follows: in addition to the dividend $D_t$, each agent receives an endowment $Y_t$ of perishable consumption goods. Therefore the total supply of consumption goods in the economy is calculated by the feasibility condition $C_t = Y_t + 2D_t$. Following the traditional setting encountered in the asset-pricing literature, dividend and endowment growth rates follow $i.i.d.$ lognormal processes

\[
\frac{D_t}{D_{t-1}} = a \epsilon^d_t, \log \epsilon^d_t \sim iiN\left(-\frac{s^2_d}{2}, s^2_d\right)
\]

\[
\frac{C_t}{C_{t-1}} = a \epsilon^c_t, \log \epsilon^c_t \sim iiN\left(-\frac{s^2_c}{2}, s^2_c\right)
\]

The endowment and dividend growth rates share the same mean $a$, and $(\log \epsilon^d_t, \log \epsilon^c_t)$ are jointly normally distributed with a correlation of $\rho_{c,d}$, and the standard deviations of $s_d$ and $s_c$. The economy is populated by a unit mass of infinite-horizon agents. We model each agent $i \in [0,1]$ to have the same standard time-separable CRRA utility function and the same subjective beliefs\(^2\). This is, however, not common knowledge among agents. The specification of agent $i$’s expected life-time utility function is

\[
E^P \sum_{t=0}^{\infty} \delta^t \frac{(C^i_t)^{1-\gamma}}{1-\gamma}
\]

\(^2\)We here adopt a representative agent model to maintain parsimony since we have shown in section 3 that heterogeneities across agents can not generate any price difference.
where $C^i_t$ is the consumption profile of agent $i$, $\delta$ denotes the discount factor of time, and $\gamma$ is the risk-aversion parameter. Instead of the objective probability measure, expectation is formed using the subjective probability measure $\mathcal{P}$, which describes the probability distributions for all external variables. Section 5.2 contains more details about the probability space. An agent’s choices are subjected to a standard budget constraint as follows:

$$C^i_t + P^A_t S^{A,i}_t + P^H_t S^{H,i}_t = (P^A_t + D_t) S^{A,i}_{t-1} + (P^H_t + D_t) S^{H,i}_{t-1} + Y_t$$

where $S^{A,i}_t$, $S^{H,i}_t$, $P^A_t$ and $P^H_t$ are defined as section 3. To avoid Ponzi schemes and to insure the existence of a maximum value, the following bounds are assumed to hold:

$$\underline{S} \leq S^{A,i}_t \leq \overline{S}$$

$$\underline{S} \leq S^{H,i}_t \leq \overline{S}$$

where the bounds $\underline{S}$ and $\overline{S}$ are assumed to be finite.

### 5.2 Probability Space

This subsection explicitly describes the general joint-probability space of the external variables. In the competitive economy, each agent considers the joint processes of endowment, dividends, and stock prices $\{Y_t, D_t, P^A_t, P^H_t\}$ as exogenous to their decision-making process. Rational expectations imply that agents know the mapping from the history of endowments $Y_t$ and dividends $D_t$ to the equilibrium of the stock prices $P^A_t$ and $P^H_t$. Stock prices, in this case, only carry redundant information. However, if the rational expectation assumption is relaxed (Adam and Marcet 2011) such that agents do not know the mapping because of no common knowledge on agents’ identical preferences and beliefs, then the equilibrium stock prices of $P^A_t$ and $P^H_t$ should be included in the underlying state space. We define the probability space as $(\mathcal{P}, \mathcal{B}, \Omega)$ with $\mathcal{B}$ denoting the corresponding $\sigma$-Algebra of the Borel subsets of $\Omega$ and $\mathcal{P}$ denoting the agent’s subjective probability measure over $(\mathcal{B}, \Omega)$. The
state space $\Omega$ for the realized exogenous variables is:

$$\Omega = \Omega_Y \times \Omega_D \times \Omega_{PA} \times \Omega_{PH}$$

where $\Omega_X$ represents the state space for all possible infinite sequences of the variable $X \in \{Y, D, PA, PH\}$. Thereby, a specific element in the set $\Omega$ represents an infinite sequence $\omega = \{Y_t, D_t, P_{A_t}, P_{H_t}\}_{t=0}^{\infty}$. The expected utility with the probability measure $P$ is defined as:

$$E_P^0 \sum_{t=0}^{\infty} \delta_t \frac{(C_t^i)^{1-\gamma}}{1-\gamma} = \int \sum_{t=0}^{\infty} \delta_t \frac{C_t^i(\omega)^{1-\gamma}}{1-\gamma} d(\omega)$$

Agent $i$ develops contingent plans for the endogenous variables $C_t^i, S_t^{A,i}, S_t^{H,i}$ according to the following policy function:

$$(C_t^i, S_t^{A,i}, S_t^{H,i}) : \Omega^t \rightarrow R^3$$

where $\Omega^t$ represents the set of histories from period zero up to period $t$.

### 5.3 Optimality Conditions

Since the objective function is concave and the feasible set is convex, the agent’s optimal plan is characterized by the first order conditions:

$$(C_t^i)^{-\gamma} P_{tA}^i = \delta E_t^P ((C_{t+1}^i)^{-\gamma}(P_{t+1}^A + D_{t+1}))$$

(5)

$$(C_t^i)^{-\gamma} P_{tH}^i = \delta E_t^P ((C_{t+1}^i)^{-\gamma}(P_{t+1}^H + D_{t+1}))$$

(6)

Before exploring why subjective beliefs can explain the AH premium, we first briefly present the unique rational expectation (RE) solution given by:

$$P_{tA,RE}^i = \frac{\delta a^{1-\gamma} \rho_\epsilon}{1 - \delta a^{1-\gamma} \rho_\epsilon} D_t$$

(7)
where \( \rho = E[(\epsilon^c_{t+1})^{1-\gamma}\epsilon^d_{t+1}] = e^{\gamma(1+\gamma)\frac{s^2}{2}}e^{-\gamma\rho_c,s^2e^{-\gamma}} \). As we know, the RE solution always generates \( P^A_{t,RE} = P^H_{t,RE} \).

We now characterize the equilibrium outcome under learning conditions. According to the arguments described by Adam, Marcet and Nicolini (2016), without strict rational expectations we may obtain \( E^P[C_{t+1}^i] \neq E^P[C_{t+1}] \), even if \( C_{t+1}^i = C_{t+1} \) holds ex-post in the equilibrium. We can make similar approximations, as follows:

\[
E_t^P[(\frac{C_{t+1}^i}{C_t^i})^{-\gamma}(P_{t+1}^A + D_{t+1})] \approx E_t^P[(\frac{C_{t+1}^i}{C_t^i})^{-\gamma}(P_{t+1}^A + D_{t+1})] \tag{9}
\]

\[
E_t^P[(\frac{C_{t+1}^i}{C_t^i})^{-\gamma}(P_{t+1}^H + D_{t+1})] \approx E_t^P[(\frac{C_{t+1}^i}{C_t^i})^{-\gamma}(P_{t+1}^H + D_{t+1})] \tag{10}
\]

The following assumption provides the sufficient conditions for this to be the case\(^3\):

**Assumption** We assume that \( Y_t \) is sufficiently large and that \( E_t^P P_{t+1}^{A(H)} / D_t < \overline{M} \) for some \( \overline{M} < \infty \) so that, given finite asset bounds \( \underline{S} \) and \( \overline{S} \), the approximations (9) and (10) hold with sufficient accuracy.

We can then define the subjective expectations for the risk-adjusted stock price growth as

\[
\beta_t^A \equiv E_t^P[(\frac{C_{t+1}^i}{C_t^i})^{-\gamma}(\frac{P_{t+1}^A}{P_t^A})] \tag{11}
\]

\[
\beta_t^H \equiv E_t^P[(\frac{C_{t+1}^i}{C_t^i})^{-\gamma}(\frac{P_{t+1}^H}{P_t^H})] \tag{12}
\]

We also assume that agents know the true processes of consumption and dividend growth.

The definitions of \( \beta_t^A \) and \( \beta_t^H \) together with the two first-order conditions (5) and (6) give rise to the asset pricing equations:

\[^3\text{Adam, Marcet and Nicolini (2016) show how this assumption plays the role in details.}\]
\[ P_t^A = \frac{\delta \alpha^{1-\gamma} \beta_t^A D_t}{1 - \delta \beta_t^A} \]  
(13)

\[ P_t^H = \frac{\delta \alpha^{1-\gamma} \beta_t^H D_t}{1 - \delta \beta_t^H} \]  
(14)

From equation (13) and (14), our learning model can generate the price differences, if \( \beta_t^A \neq \beta_t^H \), even though the two values share the same dividends \( D_t \).

### 5.4 Belief-Updating Rule

This section fully specifies the subjective probability distribution \( P \), and derives the optimal belief-updating rule for the subjective beliefs \( \beta_t^A \) and \( \beta_t^H \). Following the arguments described by Adam, Marcet and Nicolini (2016) regarding the subjective perspective of agents, the processes for the risk-adjusted stock price growth in both the Shanghai and the Hong Kong markets can be modeled as the sum of a persistent and a transitory component:

\[
\left( \frac{C_t}{C_{t-1}} \right)^{-\gamma} \frac{P_t^A}{P_{t-1}^A} = b_t^A + \epsilon_t^A, \quad \epsilon_t^A \sim \text{i.i.d.} N(0, \sigma_{\epsilon_t^A}) \\
\begin{align*}
    b_t^A &= b_{t-1}^A + \xi_t^A, \quad \xi_t^A \sim \text{i.i.d.} N(0, \sigma_{\xi_t^A})
\end{align*}
\]  
(15)

\[
\left( \frac{C_t}{C_{t-1}} \right)^{-\gamma} \frac{P_t^H}{P_{t-1}^H} = b_t^H + \epsilon_t^H, \quad \epsilon_t^H \sim \text{i.i.d.} N(0, \sigma_{\epsilon_t^H}) \\
\begin{align*}
    b_t^H &= b_{t-1}^H + \xi_t^H, \quad \xi_t^H \sim \text{i.i.d.} N(0, \sigma_{\xi_t^H})
\end{align*}
\]  
(16)

where \( b_t^A \) and \( \epsilon_t^H \) are persistent components, \( \epsilon_t^A \) and \( \epsilon_t^H \) are transitory components, all innovation components are independent from each other. Hence, agents’ perceived price processes are mutually independent. Appendix A.3 considers a general joint-process model in more detail. One way to justify these processes is that they are compatible with RE. According to the equations (7) and (8), the rational expectation for the risk-adjusted price growth is
$E_t[(\frac{C_{t+1}}{C_t})^{\gamma}\frac{P_{t+1}^A}{P_t}] = E_t[(\frac{C_{t+1}}{C_t})^{\gamma}\frac{P_{t+1}^H}{P_t}] = a^{1-\gamma}\rho_c$. The previous structure encompasses the rational expectation equilibrium as a special case in which agents believe that $\sigma_{\xi,A}^2 = \sigma_{\xi,H}^2 = 0$, and assign the probability of one to $b_0^A = b_0^H = a^{1-\gamma}\rho_c$.

We allow for a non-zero variance of $\sigma_{\xi,A}^2$ and $\sigma_{\xi,H}^2$. Agents can only observe the realizations of the risk-adjusted growth (i.e., the sum of the persistent and transitory components), therefore the requirement to forecast the persistent components $b_t^A$ and $b_t^H$ engenders a filtering problem. The priors of agents’ beliefs can be centered around some initial values and are given by the equations:

$$b_0^A \sim N(\beta_0^A, \sigma_{0,A}^2)$$

$$b_0^H \sim N(\beta_0^H, \sigma_{0,H}^2).$$

The variances of the prior distributions should be arranged in order to equal a steady-state Kalman filter uncertainty about $b_t^A$ and $b_t^H$, namely:

$$\sigma_{0,A}^2 = \frac{-\sigma_{\xi,A}^2 + \sqrt{\sigma_{\xi,A}^4 + 4\sigma_{\xi,A}^2\sigma_{\xi,H}^2}}{2}$$

$$\sigma_{0,H}^2 = \frac{-\sigma_{\xi,H}^2 + \sqrt{\sigma_{\xi,H}^4 + 4\sigma_{\xi,A}^2\sigma_{\xi,H}^2}}{2}$$

Following this, agents’ posterior beliefs will be:

$$b_t^A \sim N(\beta_t^A, \sigma_{0,A}^2)$$

$$b_t^H \sim N(\beta_t^H, \sigma_{0,H}^2)$$

The optimal updating rule implies that the evolution of $\beta_t^A$ and $\beta_t^H$ takes a constant-gain
learning form:

\[
\beta_t^A = \beta_{t-1}^A + \alpha^A ((C_{t-1})^{-\gamma} P_{t-1}^A - \beta_{t-1}^A)
\]  

\[
\beta_t^H = \beta_{t-1}^H + \alpha^H ((C_{t-1})^{-\gamma} P_{t-1}^H - \beta_{t-1}^H)
\]

where \(\alpha^A = \frac{\sigma^2_{0,A} + \sigma^2_{\xi,A}}{\sigma_{0,A}^2 + \sigma_{\xi,A}^2} \) and \(\alpha^H = \frac{\sigma^2_{0,H} + \sigma^2_{\xi,H}}{\sigma_{0,H}^2 + \sigma_{\xi,H}^2} \) are given by optimal (Kalman) gain.

The adaptive learning schemes in the equations (17) and (18), as well as the pricing equations (13) and (14) can generate rich stock-price dynamics arising from the feedback channel between the stock price \(P_t^{A(H)}\) and the subjective beliefs \(\beta_t^{A(H)}\). According to the equations (13) and (14), a high (low) \(\beta_t^{A(H)}\) will lead to a high (low) realized stock price. This will reinforce the subjective beliefs, inducing an even higher (lower) \(\beta_{t+1}^{A(H)}\) throughout the equations (17) and (18), thus leading to a much higher (lower) stock price and so on. The difference between \(\beta_t^A\) and \(\beta_t^H\) can first be generated from a difference in initial beliefs or in the learning speeds \(\alpha^A\) and \(\alpha^H\); through the self-referential property of the model, this difference in beliefs promises to generate persistent price differences between A-shares and H-shares.

Finally, in order to avoid an explosion in stock prices \(P_t^{A(H)}\), agents’ subjective beliefs \(\beta_t^{A(H)}\) are replaced by \(\omega(\beta_t^{A(H)})\), which represents the differential projection facility displayed in more detail in Appendix A.6.

5.5 Testing for the Rationality of Price Expectation

In this section we use a set of testable restrictions deducted from the agents’ beliefs system developed by Adam, Marcet and Nicolini (2016). These restrictions are listed as follows:

We establish \(x_t = (e_t, D_t/D_{t-1}, C_t/C_{t-1})\), where \(e_t \equiv \Delta(C_{t-1})^{-\gamma} P_{t-1}^H\), with \(\Delta\) denoting the first difference operator.

Restriction 1: \(E(x_{t-i} e_t) = 0\) for all \(i \geq 2\),

Restriction 2: \(E((\frac{D_t}{D_{t-1}} + \frac{D_{t-1}}{D_{t-2}} - \frac{C_t}{C_{t-1}} - \frac{C_{t-1}}{C_{t-2}}) e_t) = 0\),

Restriction 3: \(b'_{DC} \sum DC b_{DC} + E(e_t e_{t-1}) < 0\),
Restriction 1 using $\frac{D_{t+1}}{D_{t+1-1}}$  
Restriction 1 using $\frac{C_{t+1}}{C_{t+1-1}}$  
Restriction 1 using $\Delta(\frac{C_{t+1}}{C_{t+1-1}})^{-\gamma} \frac{P_{t+1}}{P_{t+1-1}}$  
Restriction 2  
Restriction 3  
Restriction 4

<table>
<thead>
<tr>
<th>Restriction</th>
<th>Test Statistics A (H)</th>
<th>5% Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Restriction 1 using $\frac{D_{t+1}}{D_{t+1-1}}$</td>
<td>2.81 (0.76)</td>
<td>9.48</td>
</tr>
<tr>
<td>Restriction 1 using $\frac{C_{t+1}}{C_{t+1-1}}$</td>
<td>4.02 (4.77)</td>
<td>9.48</td>
</tr>
<tr>
<td>Restriction 1 using $\Delta(\frac{C_{t+1}}{C_{t+1-1}})^{-\gamma} \frac{P_{t+1}}{P_{t+1-1}}$</td>
<td>2.13 (2.55)</td>
<td>9.48</td>
</tr>
<tr>
<td>Restriction 2</td>
<td>0.04 (0.15)</td>
<td>5.99</td>
</tr>
<tr>
<td>Restriction 3</td>
<td>-3.55 (-3.60)</td>
<td>1.64</td>
</tr>
<tr>
<td>Restriction 4</td>
<td>0.002 (0.001)</td>
<td>3.84</td>
</tr>
</tbody>
</table>

Table 3: Testing Subjective Beliefs against Actual Data

Restriction 4: $E(e_t) = 0,$

where $\sum_{DC} \equiv var(\frac{D_{t+1}}{D_{t+1-1}}, \frac{C_{t+1}}{C_{t+1-1}})$ and $b_{DC} \equiv \sum_{DC}^{-1} E((\frac{D_{t+1}}{D_{t+1-1}}, \frac{C_{t+1}}{C_{t+1-1}})' e_t).$

These four restrictions are necessary and sufficient conditions for the agents’ beliefs to be compatible with \( \{x_t\} \) in terms of second-order moments. Adam, Marcet and Nicolini (2016) show that, under standard assumptions, any process satisfying these testable restrictions can - in terms of its autocovariance function - be generated by the postulated system of beliefs, as seen in the equations (15) and (16). The set of derived restrictions thus fully characterizes the second-moment implications of the beliefs system. We tested the derived restrictions against the data to see if an agent’s belief system is compatible with the actual data. Table 3 reports the statistics when testing Restrictions 1-4 using the actual data.

The 5% critical values of the test statistics are reported in the last column of Table 3. For restriction 1, we compute the risk-adjusted consumption growth value for the data with $\gamma = 5$. The table shows that the test statistics are in all cases below their critical value and often by a wide margin. It then follows that agents find the observed financial data to be compatible with their belief system in terms of second moments. Based on this, we can conclude that the agents’ belief system is reasonable: Given the behavior of the actual data, the belief system is one that agents could have reasonably held.
Table 4: Parameters Values for Learning Model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>5</td>
</tr>
<tr>
<td>$\sigma_{\Delta D/D}$</td>
<td>0.0204</td>
</tr>
<tr>
<td>$\sigma_{\Delta C/C}$</td>
<td>0.0025</td>
</tr>
<tr>
<td>$a$</td>
<td>1.0014</td>
</tr>
<tr>
<td>$\rho_{c,d}$</td>
<td>-0.03</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.999</td>
</tr>
<tr>
<td>$1/\alpha^A$</td>
<td>0.0030</td>
</tr>
<tr>
<td>$1/\alpha^H$</td>
<td>0.0015</td>
</tr>
</tbody>
</table>

5.6 Quantitative Performance

This section presents the simulation outcomes for our learning model. We simulate our model at weekly frequency. We first assign a value to the coefficient of relative risk-aversion, namely $\gamma$ at 5, then calibrate the mean and standard deviation of the dividend growth $a$, $\sigma_{\Delta D/D}$, the standard deviation of the consumption growth $\sigma_{\Delta C/C}$, and the correlation between consumption growth and dividend growth $\rho_{c,d}$ using the data on the Shanghai stock market and on Chinese consumption per capita. We also calibrate $\delta$ to match the 4% annual interest rate. Meanwhile, we use the method of simulated moments to estimate $\alpha^A$ and $\alpha^H$ matching the mean, standard deviation and persistence of the AH premium. The estimation results show that $\alpha^A > \alpha^H$, which implies that agents perceive $\frac{\sigma_{\xi,A}}{\sigma_{\epsilon,A}} > \frac{\sigma_{\xi,H}}{\sigma_{\epsilon,H}}$; hence they perceive there is a market difference. Intuitively, if agents believe that the ratio of standard deviations of the persistent component shock to the transitory component shock is relatively larger for A-share prices than H-share prices, agents will tend to learn faster for A-share prices. This is because only the persistent component provides useful information for forecasting. The realized dynamics of $P_t^A$ and $P_t^H$ also support this inequality when we use the maximum likelihood estimation (MLE) method to estimate the related parameters given that data follows processes (15) and (16). Hence, it is reasonable for agents to perceive $\frac{\sigma_{\xi,A}}{\sigma_{\epsilon,A}} > \frac{\sigma_{\xi,H}}{\sigma_{\epsilon,H}}$, and the consistency between the perceived and realized data is a result of the self-referential property of the model. Table 4 contains the parameter values for the model.
We run a Monte-Carlo simulation of the learning model for $K = 10,000$ samples, with each sample having $T = 100$ periods to match the sample period of almost 2 years, namely since November 2014. Table 5 contains the simulation results. Column 2 shows the data moments of the AH premium, and Column 3 reports the 95% interval of the model’s simulated moments. We find that the mean and standard deviation of the data are located within the interval, although the model generates a slightly more persistent AH premium than the data. Figure 5 also depicts one simulation for the dynamics of the A-share price $P_t^A$ and H-share price $P_t^H$, while Figure 6 depicts the corresponding simulated AH premium. We set the initial conditions $\beta_1^A = \beta_1^H$ and $\beta_2^A$ slightly larger than $\beta_2^H$, which is more consistent with data observations. A higher learning speed $\alpha^A$ for A-share prices leads to $P_t^A$ fluctuating more relative to $P_t^H$, despite that two price dynamics retaining the same shape as shown in Figure 5, similar to the realized prices in Figure 2. When comparing Figure 6 with Figure 1, the model’s simulated AH premium index display the shape similar to the data, in that it starts at around 100, then persistently increases to about 150, while finally decreasing to about 120 after 2 years. This shows that our learning model is significantly better at generating a data-like AH premium when compared to the models in Section \(A\) and \(B\). The different learning speeds $\alpha^A$ and $\alpha^H$ by investors in two markets caused by the perceived market difference characterized by $\frac{\sigma_{\epsilon, A}}{\sigma_{\epsilon, A}} > \frac{\sigma_{\epsilon, H}}{\sigma_{\epsilon, H}}$ is the key in producing price differences. If there was no any perceived market difference\(^4\), there would be no price difference in connected markets.

Appendix A.4 presents an extended internal rationality learning model which can explain the AH premium both in the segmentation and in the connection contexts, from 2006 to 2016.

\(^4\)Another market difference could be that there are some rational bubbles existing in A-share market, not in H-share market. In Appendix A.5 we also show whether the rational bubbles can generate sufficient price differences.
<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E\left( \frac{P_A}{P_H} \right) \times 100$</td>
<td>130.71</td>
<td>[97.23 135.58]</td>
</tr>
<tr>
<td>$\sigma\left( \frac{P_A}{P_H} \right) \times 100$</td>
<td>8.97</td>
<td>[2.99 22.60]</td>
</tr>
<tr>
<td>$\rho\left( \frac{P_A}{P_H} \right) \times 100$</td>
<td>0.83</td>
<td>[0.89 0.99]</td>
</tr>
</tbody>
</table>

Table 5: Model Simulated Moments

Figure 5: Simulated Stock Prices for A-Share and H-Share

Figure 6: Simulated AH Premium Index
6. The Convergence Traders’ Strategy

In this section, we investigate the implication for convergence trader. A typical convergence trader’s strategy is to bet that the price differences between two assets with identical or similar fundamentals will narrow in the future. The convergence traders would hold long positions in an asset they consider undervalued and short positions in the other asset they consider overvalued. A famous example is the hedge fund Long-Term Capital Management (LTCM) that expected the convergence of bond yields in emerging market countries and the US (Edwards, 1999). The firm bought bonds from emerging markets and short-sold US government bonds. The spread of bond yields widened, however, because of the deterioration of the Asian financial crisis and the default of the Russian Sovereign debt. The unexpected widening led to the near-collapse of LTCM. Besides the case of LTCM, Chinese finance newspaper reported in June 2015 that many convergence traders participating in the AH-share market by short selling A-shares and long buying H-shares encountered a great loss in the end.

Xiong (2001) studied the convergence-trading strategy in a model which has three types of traders: noise traders, convergence traders and long-term traders. He found that convergence traders reduces asset price volatility in general; when an unfavorable shock causes them to suffer substantial capital losses, they liquidate their positions, thereby amplifying the original shock. This section considers the convergence traders with zero measure who take the stock prices set by learning agents as given; it then investigates the probability distribution of profits when the convergence-trading strategy is adopted.

In period 100 corresponding to 2 years since November 2014, the convergence traders expect the AH premium to narrow in the future, hence they short sell 1 unit of the A-share stock and use the income from this sale to buy H-share stock. To implement short selling in Chinese stock market, convergence traders should have approximately 50% of the short
selling value in their accounts as a security deposit. The guarantee ratio $g_{rt}$ is defined as:

$$g_{rt} = \frac{0.5 \cdot P_{100}^A + P_{t}^H \cdot \frac{P_{100}^A}{P_{100}^H}}{P_{t}^A}$$

where $P_{t}^H \cdot \frac{P_{100}^A}{P_{100}^H}$ is the market value of H-shares bought through unloading 1 unit of an A-share. If the guarantee ratio is below 130% ($i.e. g_{rt} < 130\%$), then convergence traders will experience a margin call, which could force them to liquidate part of their position at a highly unfavorable moment, and thus either suffer a loss or increase their security deposit.

The maximum duration of short selling in China is 1 year according to the regulation. We run a Monte-Carlo simulation for the learning model of 10,000 paths, with each path starting from 100th period to the 152nd period. The probability of $g_{rt} < 130\%$ can be as high as 16.7%. Liquidation happens because when prices persistently diverges and convergence traders do performance-based arbitrage they would have no enough capital to increase security deposit (Shleifer and Vishny 1997). We also calculate the distribution of profits $m_t = P_{t}^H \cdot \frac{P_{100}^A}{P_{100}^H} - P_{t}^A$ when $t = 113, 126, 139$ and 152, corresponding to 3 months, 6 months, 9 months, and 1 year. Table 6 shows the results of this simulation. We find that, even though the mean of $m_t$ is positive, it is much smaller than the standard deviation. Figure 7 shows the distribution of profit after 1 year, the fat-tail of which implies a high probability of great loss. In contrast to Xiong (2001), our learning model cannot guarantee the convergence of AH premium in the short-run. Furthermore, De Jong, Rosenthal and Van Dijk (2009) mention that the uncertainty faced by a convergence trader arises from the absence of an identifiable date at which dual-listed stock prices will converge. Therefore, it is not surprising that convergence traders have a high probability of suffering loss.
<table>
<thead>
<tr>
<th>$m_t$</th>
<th>Mean</th>
<th>Std</th>
<th>$\Pr(m_t &lt; 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3m</td>
<td>0.817</td>
<td>10.657</td>
<td>0.384</td>
</tr>
<tr>
<td>6m</td>
<td>0.662</td>
<td>17.345</td>
<td>0.365</td>
</tr>
<tr>
<td>9m</td>
<td>0.713</td>
<td>23.231</td>
<td>0.338</td>
</tr>
<tr>
<td>1y</td>
<td>0.887</td>
<td>26.659</td>
<td>0.323</td>
</tr>
</tbody>
</table>

Table 6: The Statistics of Profits from Convergence Trading Strategy

Figure 7: The Distribution of Profits by Implementing Convergence Trading Strategy (1 year)
7. Conclusion

This paper uses the AH premium as a case study to investigate whether only realized or also perceived fundamentals can influence stock prices. We have shown that present-value asset pricing models, where only fundamentals matter, regardless of the heterogeneities across agents or of possible financial market frictions, cannot generate an AH premium that fits the data sufficiently. We then proposed an internal rationality learning model, in which agents do not know the price mapping from the fundamentals to the stock prices, and in which present-value formulas do not hold, to square with the data. Hence, agents would have their own subjective beliefs about future capital gains in the two markets, and they would optimally update these beliefs. The different subjective beliefs caused by the perceived market differences allow the learning model to successfully generate an AH premium which fits the data. Finally, we showed that convergence traders with a short strategy in Shanghai and a long in Hong Kong would be very likely to suffer a great loss.

Since short selling is costly in stock markets, especially in mainland China, there are a relative small proportion of investors involved in this type of trading. As pointed out by Shleifer and Vishny (1997), when only specialized arbitrageurs accounting for a small fraction of investors implement the performance-based arbitrage, such arbitrage will not be effective in bringing stock prices to their fundamental values. However as the classical literature shows (e.g. Fama 1965, Sharpe 1964), if the number of tiny risk-neutral arbitrageurs is large, their collective actions should force stock prices to converge to their fundamental values. While we assume a zero measure of convergence traders as a shortcut in Section 6, it would be interesting to analyze the theoretical case in which there are both internal rational learning agents and a positive measure of speculators with rational price expectations. Furthermore, it appears of interest to study the implications of lowering the short selling cost in such environment. We recommend them for future research.
References


Appendix (For Online Publication)

A.1 Algorithms for State-Contingent Bond Positions

A.1.1 Rational Expectation

Step 1: Draw $N$ series of $T$ periods each of dividends $\{\{D_{n,i}\}_{i=0}^{T}\}_{n=1}^{N}$ using a $i.i.d$ random number generator. Guess a value for $\lambda$ representing constant marginal rate of substitution in the complete market and simulate for consumption $\{\{C_{1}^{1,n}, C_{2}^{2,n}\}_{i=0}^{T}\}_{n=1}^{N}$ using commodity goods market-clearing condition given initial bond holding $B_{-1}$. Solve for $\lambda$ by iteration using the intertemporal life time budget constraint for type 1 agent as follows:

$$\frac{1}{N} \sum_{n=1}^{N} \sum_{i=0}^{T} \delta^{j,i} \frac{u'(C_{t})}{u'(C_{0})} (C_{t+j}^{1} - D_{t+j}^{1}) = B_{-1}^{1}$$

Step 2: Draw one series of $T_{L}$ periods of dividends using a $i.i.d$ random number generator. Find the corresponding consumption $\{C_{1}^{1}, C_{2}^{2}\}_{i=0}^{T_{L}}$ by the market-clearing condition given the solution for $\lambda$. Find the present value of primary deficit denoted by $Dd_{t}^{1} \equiv \sum_{j=0}^{\infty} \delta^{1,j} \frac{u'(C_{t}^{1})}{u'(C_{0}^{1})} (C_{t+j}^{1} - D_{t+j}^{1})$ for agent 1, which can be solved backwards assuming $Dd_{T_{L}}^{1} = 0$.

Step 3: We solve for state-contingent-bond positions for agent 1 $\{B_{t-1}^{1}(D)\}_{i=0}^{T_{L}}$ by using the equation:

$$B_{t-1}^{1}(D^{j}) = E(Dd_{t}|D_{t} = D^{j})$$

where bond holding are just function of state $D$. In practice we use average over time to obtain the bond positions:

$$B_{t-1}^{1}(D^{j}) = \frac{1}{T} \sum_{t=1}^{T} Dd_{t}^{1} I_{j}(D_{t})$$

where $I_{j}(D_{t})$ is the indicator function taking value of 1 if $D^{j}$ happens in period $t$ i.e. $D_{t} = D^{j}$ for $j \in \{h,l\}$. This could also be regarded as running a regression of $Dd$ on indicator functions as follows:

$$Dd_{t}^{1} = \alpha_{h}^{1} I_{h}(D_{t}) + \alpha_{l}^{1} I_{l}(D_{t}).$$
Type 2 Agents’ bond positions are just the opposite of type 1 agents’.

A.1.2 Diverse Belief

In this case, every step is same as in rational expectation case except that the marginal rate of substitution $\lambda$ is not constant any more because of the diverse beliefs. The time-varying marginal rate of substitution $\{\lambda_t\}$ follows:

$$\lambda_t = \alpha_{t-1}(D_t)\lambda_{t-1}$$

where $\alpha_{t-1}(D_t) = \frac{\text{prob}^2(D_t)}{\text{prob}^1(D_t)}$. Therefore, the state-contingent-bond positions are not only the function of $D$ but also a function of $\lambda_{t-1}$ as follows:

$$B_{t-1}^1(D^j) = E(Dd_t|D_t = D^j, \lambda_{t-1})$$

Run a regression of $Dd$ on both $D$ and $\lambda_{t-1}$ leads to simulation of state-contingent-bond positions for agent 1, which can be expressed as:

$$Dd_t^1 = (\alpha_0^h + \alpha_1^h\lambda_{t-1})I_h(D_t) + (\alpha_0^l + \alpha_1^l\lambda_{t-1})I_l(D_t)$$

As is well known, the regression can be run separately for high and low dividends.

A.2 Agree to Disagree

Here we provide the proof that in the complete market environment we can use either agent’s SDF to price asset even though they have diverse beliefs related to fundamentals.
\[ P_{t}^{1,A(H)} = E_t^1 \sum_{j=1}^{\infty} (\delta^1)^j \prod_{k=1}^{j} \frac{u_c^1(C_{1+k}^1)}{u_c^1(C_{1+k-1}^1)} D_{t+j} \]

\[ = E_t^1 \sum_{j=1}^{\infty} (\delta^1)^j \prod_{k=1}^{j} \frac{u_c^2(C_{1+k}^2)}{u_c^2(C_{1+k-1}^2)} A_{t+j} D_{t+j} \]

\[ = \sum_{j=1}^{\infty} [v(\delta^2)^j \prod_{k=1}^{j} \frac{u_c^2(C_{1+k}^2)}{u_c^2(C_{1+k-1}^2)} D_{t+j} + (1-v)(\delta^2)^j \prod_{k=1}^{j} \frac{u_c^2(C_{1+k}^2)}{u_c^2(C_{1+k-1}^2)} D_{t+j}] \]

\[ = E_t^2 \sum_{j=1}^{\infty} (\delta^2)^j \prod_{k=1}^{j} \frac{u_c^2(C_{1+k}^2)}{u_c^2(C_{1+k-1}^2)} D_{t+j} \]

\[ = P_{t}^{2,A(H)} \]

### A.3 Cross-Learning Scheme

Instead of modeling agents’ belief system independent of each other as (15) and (16) proceed, it is also natural to model it as the cross-learning scheme, since agents probably think that two shares of the same fundamentals should be correlated. The belief system is expressed as follows:

\[
\begin{bmatrix}
    \frac{P_{t}^A}{P_{t-1}^A} \\
    \frac{P_{t}^H}{P_{t-1}^H}
\end{bmatrix} = \begin{bmatrix} b_{t}^A \\ b_{t}^H \end{bmatrix} + \begin{bmatrix} \epsilon_{t}^A \\ \epsilon_{t}^H \end{bmatrix}
\]

\[
\begin{bmatrix}
    b_{t}^A \\
    b_{t}^H
\end{bmatrix} = \begin{bmatrix} b_{t-1}^A \\ b_{t-1}^H \end{bmatrix} + \begin{bmatrix} \xi_{t}^A \\ \xi_{t}^H \end{bmatrix}
\]

\((\epsilon_{t}^A, \epsilon_{t}^H) \sim N(0, R)\) and \((\xi_{t}^A, \xi_{t}^H) \sim N(0, Q)\), where \(R\) and \(Q\) could be non-diagonal variance-covariance matrices. The independent belief system is a special case of the above setting in which \(R\) and \(Q\) are diagonal matrices. Define \(X\) and \(K\) by \(X(X + R)^{-1}X = Q\) and \(K = X(X + R)^{-1}\). \(X\) satisfies the Ricatti equation in the steady state and \(K\) is the Kalman
gain vector. Agents optimally update their beliefs according to:

\[
\begin{bmatrix}
\beta^A_t \\
\beta^H_t
\end{bmatrix} = 
\begin{bmatrix}
\beta^A_t \\
\beta^H_t
\end{bmatrix} + 
\begin{bmatrix}
\alpha^A & \alpha^C \\
\alpha^C & \alpha^H
\end{bmatrix} 
\begin{bmatrix}
(C_{t-1} - C_{t-2}) - \gamma \frac{P^A_{t-1}}{P^A_{t-2}} - \beta^A_{t-1} \\
(C_{t-1} - C_{t-2}) - \gamma \frac{P^H_{t-1}}{P^H_{t-2}} - \beta^H_{t-1}
\end{bmatrix}
\]

Quantitatively, we estimate the parameters \([\alpha^A, \alpha^H, \alpha^C]\) to match the mean, standard deviation, persistence of AH premium and the correlation of A-share price and H-share price in the whole sample. The estimated parameters are \(\alpha^A = 0.0032, \alpha^H = 0.0015, \alpha^C = 0.0001\). The Monte-Carlo simulation results are in the Table 7.

### A.4 A Learning Model Covering Segmentation and Connection Episodes

Figure 8 shows the historical changes of AH premium from 2006 to 2016. We see that high AH premium is not only the phenomena during the connection periods, but also in the whole sample. In this section, we show that a modified internal rationality learning model can produce the AH premium in the whole sample, even though the focus of our paper is the price differences in connected markets.

We extend the benchmark learning model to cover both segmentation and connection periods. The dividend and consumption growths still follow the same processes as in Section 5. We only present how this case differs from the case in the main text.
Figure 8: The Historical Changes in AH Premium from 2006 to 2016

The maximization problem for agent 1 in this case is

$$\max_{\{C_t^1, S_t^{A,1}, S_t^{H,1}\}} E_t^p \sum_{t=0}^{\infty} \delta^t (C_t^1)^{1-\gamma}$$

$$s.t. C_t^1 + P_t^A S_t^{A,1} = (P_t^A + D_t) S_{t-1}^{A,1} + Y_t \text{ if } t \leq N$$

$$C_t^1 + P_t^A S_t^{A,1} + P_t^H S_t^{H,1} = (P_t^A + D_t) S_{t-1}^{A,1} + (P_t^H + D_t) S_{t-1}^{H,1} + Y_t \text{ if } t \geq N$$

$$0 \leq S_t^{A,1} \text{ & } 0 \leq S_t^{H,1}$$

For each period, first-order conditions for type 1 agent become:

$$C_t^1 : (C_t^1)^{-\gamma} - \lambda_t^1 1(t \leq N) - \eta_t^1 1(t > N) = 0$$

$$S_t^{A,1} : -\lambda_t^1 1(t \leq N) P_t^A - \eta_t^1 1(t > N) P_t^A$$

$$+ E_t^p [\lambda_{t+1}^1 1(t \leq N)(P_{t+1}^A + D_{t+1}) + \eta_{t+1}^1 1(t > N)(P_{t+1}^A + D_{t+1})] \leq 0$$

$$S_t^{H,1} : -\eta_t^1 1(t > N) P_t^H + E_t^p [\eta_{t+1}^1 1(t > N)(P_{t+1}^H + D_{t+1})] \leq 0$$
where \( \lambda^1_t \) and \( \eta^1_t \) are Lagrangian multipliers for the above budget constraints, \( 1(\cdot) \) is the indicator function. The Euler equations in this case are:

\[
(C^1_t)^{-\gamma} P^A_t \geq \delta E^p_t \left( (C^1_{t+1})^{-\gamma}(P^A_{t+1} + D_{t+1}) \right) \text{ with equality if } S^A_{t+1} > 0
\]

\[
(C^1_t)^{-\gamma} P^H_t \geq \delta E^p_t \left( (C^1_{t+1})^{-\gamma}(P^H_{t+1} + D_{t+1}) \right) \text{ with equality if } S^H_{t+1} > 0 \text{ and } t > N
\]

Similarly, the maximization problem for type 2 agent is:

\[
\max E^p_0 \sum_{t=0}^{\infty} \delta^t (C^2_t)^{1-\gamma} \frac{1}{1-\gamma} \text{ s.t. } C^2_t + P^H_t S^A_{t+1} = \left( (P^H_t + D_t) S^H_{t-1} + Y_t \right) \text{ if } t \leq N \quad (20)
\]

\[
C^2_t + P^A_t S^A_{t+1} + P^H_t S^H_{t+1} = \left( (P^A_t + D_t) S^A_{t-1} + (P^H_t + D_t) S^H_{t-1} + Y_t \right) \text{ if } t \geq N
\]

For each period, first-order conditions for agent 2 are:

\[
C^2_t : (C^2_t)^{-\gamma} - \lambda^2_t 1(t \leq N) - \eta^2_t 1(t > N) = 0
\]

\[
S^H_{t+1} : -\lambda^2_t 1(t \leq N) P^H_t - \eta^2_t 1(t > N) P^H_t
\]

\[
+ E^p_t \left[ \lambda^2_{t+1} 1(t \leq N)(P^H_{t+1} + D_{t+1}) + \eta^2_{t+1} 1(t > N)(P^H_{t+1} + D_{t+1}) \right] \leq 0
\]

\[
S^A_{t+1} : -\eta^2_t 1(t > N) P^A_t + E^p_t \left[ \eta^2_{t+1} 1(t > N)(P^A_{t+1} + D_{t+1}) \right] \leq 0
\]

And the Euler equations are:

\[
(C^2_t)^{-\gamma} P^A_t \geq \delta E^p_t \left( (C^2_{t+1})^{-\gamma}(P^A_{t+1} + D_{t+1}) \right) \text{ with equality if } S^A_{t+1} > 0 \text{ and } t > N
\]
Moments | Data | Model |
<table>
<thead>
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<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>$E(\frac{P_A}{P_H} * 100)$</td>
<td>118.13</td>
<td>[97.44 121.48]</td>
</tr>
<tr>
<td>$\sigma(\frac{P_A}{P_H} * 100)$</td>
<td>21.79</td>
<td>[5.77 25.88]</td>
</tr>
<tr>
<td>$\rho(\frac{P_A}{P_H} * 100)$</td>
<td>0.97</td>
<td>[0.97 0.99]</td>
</tr>
</tbody>
</table>

Table 8: Model Simulated Moments

$$(C_t^2)^{-\gamma} P_t^H \geq \delta E_t^{P^2} ((C_{t+1}^2)^{-\gamma} (P_{t+1}^H + D_{t+1})) \text{ with equality if } S_t^{H,2} > 0$$

The consumption goods market-clearing condition becomes:

$$C_t = C_t^1 + C_t^2 = 2Y_t + 2D_t$$

The pricing equations according to Adam and Marcet (2011) are:

$$P_t^A = \left\{ \begin{array}{ll}
\frac{\delta(a)^{1-\gamma} \rho_a}{1-\delta \beta_t^{L^2}} D_t & \text{if } t \leq N \\
\max_{i=1,2} \frac{\delta(a)^{1-\gamma} \rho_a}{1-\delta \beta_t^{L^2}} D_t & \text{if } t > N
\end{array} \right.$$  

$$P_t^H = \left\{ \begin{array}{ll}
\frac{\delta(a)^{1-\gamma} \rho_a}{1-\delta \beta_t^{H^2}} D_t & \text{if } t \leq N \\
\max_{i=1,2} \frac{\delta(a)^{1-\gamma} \rho_a}{1-\delta \beta_t^{H^2}} D_t & \text{if } t > N
\end{array} \right.$$  

Finally, the quantitative results are shown in Table 8. The 95% interval of model’s simulated moments contain moments for realized data. The internal rationality learning mechanism can explain both segmentation and connection AH premium.

### A.5 Rational Bubble

This section explores if the rational bubble story can fit the AH premium data using an abstract model. Without lost of generalization, the stock price in Hong Kong $P_t^H$ is assumed to have fundamental value $v_t$, which is the expected discounted value of the future dividends. The stock price in Shanghai $P_t^A$ on the contrary contains a bubble term and is the sum of
fundamental value \( v_t \) and a rational bubble component \( b_t \). That is:

\[
P_t^{\text{HH}} = v_t
\]

\[
P_t^{\text{AA}} = v_t + b_t
\]

Following the literature (Tirole, 1985), the process for bubble \( b_t \) should satisfy a no-arbitrage condition assuming constant interest rate \( r \)

\[
b_{t-1} = E_{t-1} \left[ \frac{1}{1 + r} b_t \right]
\]

We can rewrite it as:

\[
b_t = (1 + r) b_{t-1} + \varepsilon_t^b
\]

(21)

where \( \varepsilon_t^b \) is the shock to the bubble component. The price ratio is given by:

\[
\frac{P_t^{\text{AA}}}{P_t^{\text{HH}}} = \frac{v_t + b_t}{v_t}
\]

(22)

The fundamental value \( v_t \) barely varies at weekly frequency, as can be seen from the equation (7) and (8). Hence, the variation of \( \frac{P_t^{\text{AA}}}{P_t^{\text{HH}}} \) should mainly come from \( b_t \). And according to equation (21), \( b_t \) can be backward iterated and expressed as:

\[
b_t = (1 + r)^t b_0 + \sum_{i=1}^{t} (1 + r)^{t-i} \varepsilon_i^b \text{ for } t \geq 1
\]

(23)

We calibrate \( r = 0.5\% \) to match Chinese real interest rate and \( b_0 = 0.01 v_0 \) to match initial 1% AH premium. Given the low interest rate \( r \) and small initial value \( b_0 \), it is possible, as can be seen from equation (22) and (23), to generate data-like AH premium when some certain history of bubble shock \( \{\varepsilon_i^b\}_{i=1}^{t} \) happen. However, the results heavily rely on the exogenous bubble shocks, it is a little bit arbitrary with a lack of economic logic.
A.6 Differentiable Projection Facility

The function $\omega$ for the differentiable projection facility is:

$$
\omega(\beta) = \begin{cases} 
\beta & \text{if } x \leq \beta^L \\
\beta^L + \frac{\beta - \beta^L}{\beta^U - \beta^L}(\beta^U - \beta^L) & \text{if } \beta^L < x \leq \beta^U 
\end{cases}
$$

In our numerical exercise, we choose $\beta^U$ such that the implied price dividend ratio never exceeds $U^{PD} = 600$ and set $\beta^L = \delta^{-1} - 2(\delta^{-1} - \beta^U)$.

A.7 Data Sources

Our dataset for Chinese stock market prices, dividends, Hang Seng China AH premium index, Hang Seng China A index and Hang Seng China H index are downloaded from Wind Financial Database (http://www.wind.com.cn). The daily price series have been transformed into weekly series by taking the index value of the last days of the corresponding weeks. Our dataset for Chinese macro data, particularly consumption and CPI, are downloaded from the website for Chang et al. (2015). (http://www.tzha.net/code).