

# *Understanding AH Premium in China Stock Market*

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## **Abstract**

There are 88 companies (AH-share) dual-listed in both China mainland stock market (A-share) and Hong Kong stock market (H-share) accounted for 20% of total A-share. The ‘Shanghai-Hong Kong Stock Connect’ program starting at November, 2014 makes previously two segmented markets—Shanghai and Hong Kong stock markets—connected. The price difference of AH-share in Shanghai and Hong Kong stock markets, measured by Hang Seng China AH Premium Index, persistently divergences instead of converging, and even reaches 50% higher in Shanghai market. We have shown that present-value asset pricing models with heterogeneous agents cannot generate any price difference. Transaction cost and different dividend taxes between Shanghai and Hong Kong markets also quantitatively fails to explain such high and volatile AH premium. We, hence, propose an ‘Internal Rationality’ learning model, in which agents don’t know the pricing function from fundamentals to the stock prices and have different subjective beliefs about tomorrow’s capital gains in Shanghai and Hong Kong markets. Our learning model can successfully generate data-like weekly AH premium. We also show that convergence traders with strategy short in Shanghai and long in Hong Kong will lose money with 33% probability.

# 1 Introduction

This chapter studies the stock prices difference named AH premium in the connected China A- and H-share markets, which is an interesting anomaly in asset markets. The stocks listed in China mainland stock exchanges (Shanghai and Shenzhen) are called A-share, and the one listed in Hong Kong exchange are called H-share. There are 88 companies dual-listed in A-share and H-share markets called AH-share, which are identical with respect to shareholder rights, such as voting and profit-sharing. Most of AH-share companies are big ones, especially state-owned enterprises, accounting for 20% of total market value in A-share market. Hang Seng China AH Premium Index plotted in figure ?? measures the weighted averaged price difference of AH-share. Index equaling 100 means that A-share are trading at par with H-share, larger than 100 for A-share trading at a premium versus H-share, smaller than 100 for A-share trading at a discount versus H-share.

Figure ?? shows that AH-share prices are always different even though they have the same fundamentals in Shanghai and Hong Kong markets. Before November 2014, Shanghai and Hong Kong markets were segmented that mainland investors are not allowed to invest in Hong Kong market and either for foreign investors in Shanghai market. The price difference of dual-listed stocks in the segmented markets is widely studied in the literature. Fernald and Rogers (2002) attribute the discount of China B-share stock (only for foreigners) to A-share stock (only for the domestic) to the fact that Chinese investors have a higher discount rate than foreigners. Chan, Menkveld and Yang (2008) show the evidence that AB-share premium is caused by the fact that foreign investors, who trade B-share, have an informational disadvantage relative to domestic investors, who trade A-share. While, Mei, Scheinkman and Xiong (2009) propose that trading caused by investors' speculative motives can help explain a significant fraction of the price difference between the perfect segmented dual-class AB-share.

The previously segmented Shanghai and Hong Kong markets, however, become connected since the starting of Shanghai-Hong Kong stock connect program in November, 2014. The AH premium index should converge to 100

according to the standard present-value asset pricing theory, but it diverges dramatically to arrive at almost 150 and then fluctuates between 120 and 150. There are very few works on the price difference in the connected markets except Froot and Dabora (1999) focusing on only three twin stocks. This paper studies the price difference in the sample with 88 dual-listed stocks.

This paper first investigates whether the present-value heterogeneous asset pricing models can generate sufficiently high, volatile and persistent AH premium. The heterogeneity could be reflected in agents' different discount factors (Fernald and Rogers, 2002), diverse beliefs caused by asymmetric information (Chan, Menkveld and Yang, 2008), and different transaction costs and dividend taxes (Froot and Dabora, 1999). The model environment could be complete market or incomplete market, and stock prices equalling with the discounted sum of expected future dividends makes agents like fundamental investors. We find that different risk aversions, discount factors, and diverse beliefs cannot produce any AH premium, transaction costs are so small that could be ignored, and dividend taxes are possible to generate constant 6% premium. The generalized model we show in section 2.4 illuminates that prices for A-share and H-share in the connected markets are the same in each period when we only have variations across agents without variations across two shares.

The failure of present-value models in producing AH premium motivates us to propose an 'Internal Rationality' learning model as Adam, Marcet and Nicolini (2016), in which agents who do not know the mapping from the fundamentals to price and optimize their behaviors based on their subjective beliefs about all variables that are beyond their control. Given the subjective beliefs we specify, agents behave as speculators and optimally update their expectations about capital gains using Kalman filter. Agents' subjective expectations in turn influence equilibrium stock prices, and the realized stock prices feed back into agents' beliefs. If agents have initial different beliefs or different learning speeds between A-share and H-share, agents can have different subjective beliefs on them which generate different stock prices.

Finally we study the convergence traders' strategy, which relies on the price convergence of similar or identical assets. A typical convergence trade would short sell in AH-share in Shanghai market, and long buy it in the Hong Kong market. But the learning model shows that prices cannot converge in the short-run. Since the longest duration of short-selling tool is one-year, we calculate the distribution of money-making for 3, 6, 9 months and one year.

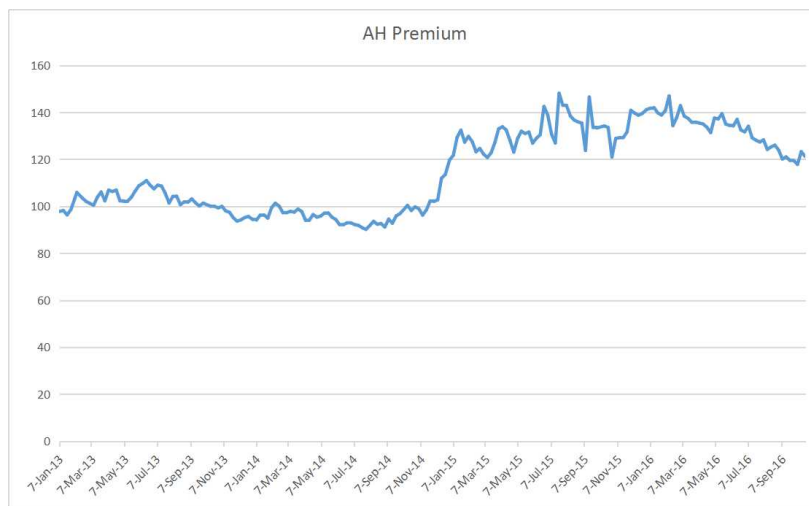


Figure 1: Hang Seng China AH Premium Index

We find that convergence traders have a large probability to lose money.

## 2 Overview of China Stock Market

China mainland stock market is relatively young and started in 1990 with the establishment of two exchanges: the Shanghai Stock Exchange (SSE) and Shenzhen Stock Exchange (SZSE). The number of listed companies was just 13 in the starting time. During the period from 1990 to 2015, the Chinese economy has performed well with averaged annual 10% GDP growth rate. The extraordinary economic growth undoubtedly leads to the rapid growth of financial markets. The market value of China stock market (excluding Hong Kong and Taiwan) reaches \$8.4 trillion at the end of 2015 which makes it the second largest one in the world, even though the ratio of market capitalization to GDP is relatively low at about 60%. The number of listed companies also rises to 2827. The main boards of the Shanghai and Shenzhen Stock Exchanges list larger and more mature stocks, like the NYSE in the US. The Shenzhen Stock Exchange also includes two other boards, the Small and Medium Enterprise Board and the ChiNext Board, also known as the Growth Enterprise Board, which provides capital for smaller and high-technology stocks, like the NASDAQ in the US. Mainland stock market has a dual-share

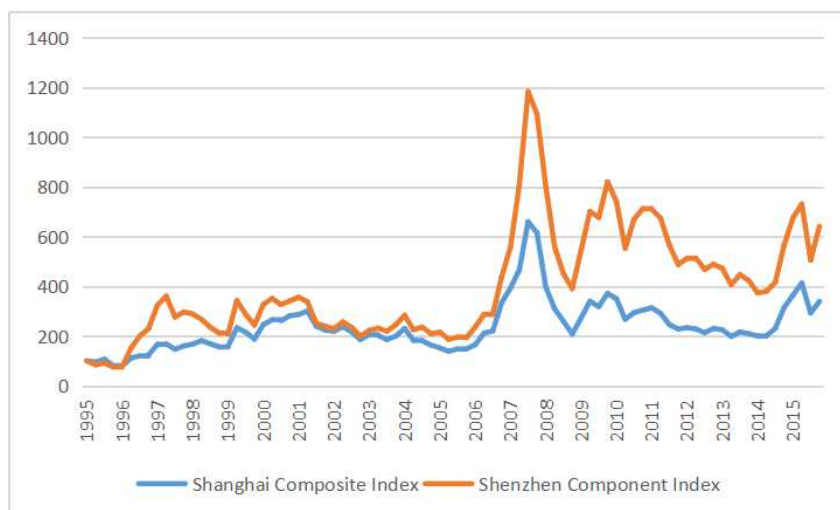


Figure 2: China Stock Price Index

system. Before the starting of Shanghai-Hong Kong Stock connect, mainland investors can invest only in A-share, while foreign investors can invest only in B-share.

Figure ?? shows the dynamics of stock prices indexes in mainland Shanghai and Shenzhen markets from 1995 to 2015 respectively. Mainland stock price experiences two episodes of obvious boom and bust, one is 2006-2007 and the other is 2014-2015. Stock prices reached the historical peak in 2007 from the bottom in 2005, then quickly busted. Then, from 2008 to 2014 the market generally trended down. Therefore, Allen et al. (2015) think that the performance of China stock market has been disappointing, especially compared with the growth of GDP. The market price boomed again in the second half of 2014, and almost doubled in the middle of 2015. One distinguished characteristic in China stock market is that stock trading is new to most of participants, 80% of them are individual investors (Mei, Scheinkman and Xiong, 2009). Given the typical Chinese investor's lack of experience, it is reasonable to hypothesize that these investors would often disagree about stock valuation and as a result would behave more like the speculators. The larger volatility in Chinese stock markets than US shown in table 1 support this.

Statistics	SSE	SZSE	S&P500	CRSP
Std.dev. stock return $\sigma_{rs}$	17.06%	22.32%	8.17%	8.69%
Std.dev. stock return $\sigma_{PD}$	277.83	167.07	47.26	54.94

Table 1: China and US Stock Market Volatility

### 3 Present-Value Models

In this section, we build present-value models and explore on the potential factors driving the price difference. We consider variations of discount factor, risk aversion, beliefs on fundamental, dividend tax and transaction cost across agents in the complete market and incomplete markets.

#### 3.1 Models in Complete Market

Let's describe the economy in the complete market. Basically it is a Lucas tree model with two type of agents.

##### 3.1.1 Rational Expectation

In the beginning we endow the agents with objective beliefs i.e. rational expectation. The type  $i$  investors in the economy account for a fraction of  $\mu^i > 0$  of population  $i \in \{1, 2\}$  respectively, where  $\mu^1 + \mu^2 = 1$ . Type 1 agent stands for mainland investor and 2 for Hong Kong investor. The two types may differ with respect to their degree of risk aversion, discount factor.

Investors' portfolio includes A-share, H-share and contingent bonds. Agents trade A-share and H-share with each other in this economy.  $S_t^{1,A}, S_t^{1,H}, S_t^{2,A}, S_t^{2,H}$  are denoted as A-and H-share stocks that agent 1 and agent 2 buy respectively on period  $t$ . One unit of A-share and H-share pay investors the same dividend as

$$D_t^A = D_t^H = D_t$$

For convenience and without loss of generality we assume the exogenous dividend process in the complete market economy is *i.i.d* taking two values of  $D_h$  and  $D_l$  at each period, where  $Prob(D_l) = \pi, Prob(D_h) = 1 - \pi$ . We start exploring on complete market with Arrow securities  $B_t(D_h)$  and  $B_t(D_l)$  that pays 1 unit of consumption if dividend payment on  $t + 1$  is high and low respectively.

Commodity goods market clearing condition is

$$2D_t = \mu^1 C_t^1 + \mu^2 C_t^2$$

Arrow securities markets clear conditions are

$$\mu^1 B_t^1(D_j) + \mu^2 B_t^2(D_j) = 0 \quad \forall j = h, l$$

A- and H-share market clearing are

$$\mu^1 S_t^{1,Z} + \mu^2 S_t^{2,Z} = 1 \quad \forall Z = A, H$$

We assume utility function is increasing, concave and continuously differentiable. The investors' maximization problem is

$$\max_{\{C_t^i, S_t^{i,A}, S_t^{i,H}, B_t\}} E_0 \sum_{t=0}^{\infty} (\delta^i)^t u_i(C_t^i)$$

$$\begin{aligned} s.t. \quad & S_t^{i,A} P_t^A + S_t^{i,H} P_t^H + C_t^i + B_t^i(D_h)Q_t(D_h) + B_t^i(D_l)Q_t(D_l) \\ & = S_{t-1}^{i,A}(P_t^A + D_t^A) + S_{t-1}^{i,H}(P_t^H + D_t^H) + B_{t-1}^i \end{aligned}$$

And we can impose no-short-selling constraints as

$$\begin{aligned} 0 &\leq S_t^{i,A} \\ 0 &\leq S_t^{i,H} \end{aligned}$$

F.O.Cs lead to

$$\delta^1 E_t \frac{u_c^1(C_{t+1}^1)}{u_c^1(C_t^1)} = \delta^2 E_t \frac{u_c^2(C_{t+1}^2)}{u_c^2(C_t^2)} \quad (1)$$

where  $u_c^i$  is marginal utility of type  $i$  agent.

This result of full insurance features complete market. Although agents could have different discount factors and risk aversions, the property of full insurance gives rise to the same stochastic discount factor (SDF) as equation (1). Given the stock prices having the present-value expression, the same dividends and SDFs produce no price difference between A-share and H-share. Therefore discount factors and different relative risk aversions across two agents are not able to drive any price difference in the connected markets.

Now we are exploring the black box that how agents arrive at full insurance through trading contingent bonds. This is not studied in the literature



	$c_h$	$c_l$	$b_h$	$b_l$
Agent 1	0.8736	0.5514	-0.0036	0.0988
Agent 2	1.1264	0.4486	0.0036	-0.0988

Table 2: Contingent Bond Holding

to our best knowledge. For illustration, we fully solve an exercise where both agents have CRRA utility with same discount factor but agent 1 is more risk averse than agent 2 by approximating expectations in Euler equations with log linear polynomials.

With state contingent bonds, stock A and stock H are indeterminate 'redundant' assets. So we can keep agents' holding of the two assets fixed over time. Intuitively agent 1 prefer smoother consumption than agent 2 does because agent 1 is more relative risk averse. The full insurance is achieved through agent 1 always buying low contingent bond and selling high contingent bond. We confirm this by numerically solving the quantity of bond holdings, and find that agent 1's consumption are relatively smoother across nature states than agent 2's while their consumption varies with endowment. These observations are shown in table 2. The algorithm in detail is in appendix 2.8.1.

Different dividend taxes are suspected to be eligible in driving the price difference. We then investigate whether dividends taxes could generate quantitatively sufficient AH premium. Hence we now add tax into the model.

$$\begin{aligned}
& S_t^{i,A} P_t^A + S_t^{i,H} P_t^H + C_t^i + B_t^i(D_h)Q_t(D_h) + B_t^i(D_l)Q_t(D_l) \\
= & S_{t-1}^{i,A}(P_t^A + (1 - \tau^{i,A})D_t^A) + S_{t-1}^{i,H}(P_t^H + (1 - \tau^{i,H})D_t^H) + B_{t-1}^i \forall i
\end{aligned}$$

In data,  $\tau^{1,A}$  is 5%,  $\tau^{1,H}$  is 20%,  $\tau^{2,A}$  is 10% and  $\tau^{2,H}$  is 10%. The agent offering higher price will be the marginal one for the security. Hence type 1 is marginal in Shanghai and type 2 is marginal in Hong Kong as can be seen in the F.O.Cs below. Without loss of generality, we assume that two agents have log utility in the following.

The F.O.Cs in this case become

$$\begin{aligned}
P_t^A &= E_t \delta^1 \frac{C_t^1}{C_{t+1}^1} [P_{t+1}^A + (1 - \tau^{1,A})D_{t+1}] \quad 0 \leq S_t^1 \\
P_t^A &> E_t \delta^2 \frac{C_t^2}{C_{t+1}^2} [P_{t+1}^A + (1 - \tau^{2,A})D_{t+1}] \quad S_t^2 = 0
\end{aligned}$$

Similarly for H share, we have

$$P_t^H > E_t \delta^1 \frac{C_t^1}{C_{t+1}^1} [P_{t+1}^H + (1 - \tau^{1,H}) D_{t+1}] \quad S_t^1 = 0$$

$$P_t^H = E_t \delta^2 \frac{C_t^2}{C_{t+1}^2} [P_{t+1}^H + (1 - \tau^{2,H}) D_{t+1}] \quad 0 \leq S_t^2$$

A-share price is the discounted sum of future dividend by type 1's SDF, so for H-share by type 2's SDF. Hence we obtain

$$P_t^A = E_t \left[ \sum_{j=1}^{\infty} (\delta^1)^j \prod_{k=1}^j \frac{C_{t+k-1}^1}{C_{t+k}^1} (1 - \tau^{1,A}) D_{t+j} \right] \quad (2)$$

$$P_t^H = E_t \left[ \sum_{j=1}^{\infty} (\delta^2)^j \prod_{k=1}^j \frac{C_{t+k-1}^2}{C_{t+k}^2} (1 - \tau^{2,A}) D_{t+j} \right] \quad (3)$$

And since the tax is constant and can be factored out. Dividing (2) by (3) leads to

$$\frac{P_t^A}{P_t^H} = \frac{1 - \tau^{1,A}}{1 - \tau^{2,H}}$$

Hence price ratio is constant over time with approximate ratio of 1.056, which contradicts with the observation that standard deviation of AH premium fluctuates between 1.2 and 1.5. Furthermore it's worthwhile to notice that if  $\tau^{1,A} = \tau^{1,H}$  and  $\tau^{2,A} = \tau^{2,H}$ , then one would notice same price of A and H shares after a quick look at the F.O.Cs. Hence if mainland investors face the same dividend tax of A share and H share and so do the foreign investors, there would be no price difference in this complete market framework even if dividends taxes are not the same across agents.

### 3.2 Diverse Belief

Furthermore, there is popular narrative in the market that says foreign investors are pessimistic about Chinese economy but mainland people have optimistic views on the contrary. These diverse beliefs may be due to imperfect information or other reasons. Dealers and market analysts tend to tell this kind of story to rationalize the AH premium. So let's see what happens

when two agents have diverse beliefs on fundamental in this environment while setting the market frictions discussed above aside. Towards this end, we depart a bit from rational expectation model in the way that two agents are endowed with diverse beliefs on fundamental.

Let's assume agent 1 is optimistic while agent 2 is pessimistic. More important let's assume agent 1 is relatively right compared to agent 2. We will see that in the complete market agent 1 will take advantage of his information superiority so that he accumulates assets and consume more goods. Formally let  $i \in \{o, p\}$  where  $o$  stands for optimistic agent and  $p$  stands for pessimistic agent. Optimistic agent perceives  $Prob(D_h) = u$  while pessimistic agent perceives  $Prob(D_l) = v$  where  $u > v$ . And the true objective probability follows

$$Prob(D_l) = \pi, Prob(D_h) = 1 - \pi$$

Let  $\mathbf{1}(D_h)$  and  $\mathbf{1}(D_l)$  be the indicator function that take value 1 if  $D_h$  and  $D_l$  happen respectively. F.O.Cs lead to

$$\begin{aligned} \frac{C_{t+1}^p}{C_{t+1}^o} &= \frac{C_t^p \delta^p v}{C_t^o \delta^o u} \mathbf{1}(D_{t+1}^h) + \frac{C_t^p \delta^p (1-v)}{C_t^o \delta^o (1-u)} \mathbf{1}(D_{t+1}^l) \\ &= \left[ \frac{v}{u} \mathbf{1}(D_{t+1}^h) + \frac{1-v}{1-u} \mathbf{1}(D_{t+1}^l) \right] \frac{\delta^p C_t^p}{\delta^o C_t^o} \end{aligned} \quad (4)$$

where  $\left[ \frac{v}{u} \mathbf{1}(D_{t+1}^h) + \frac{1-v}{1-u} \mathbf{1}(D_{t+1}^l) \right]$  is denoted as  $A_{t+1}$  for simplicity.

The assumption that agent 1 is relatively right leads to  $\left(\frac{v}{u}\right)^\pi \left(\frac{1-v}{1-u}\right)^{1-\pi} > 1$ , which implies that agent 1 will gradually consume the total dividends in the economy consistent with the conclusion in Bloom and Easley (2006). Rearranging (4) gives

$$\delta^o \frac{C_t^o}{C_{t+1}^o} = A_{t+1} \delta^p \frac{C_t^p}{C_{t+1}^p}$$

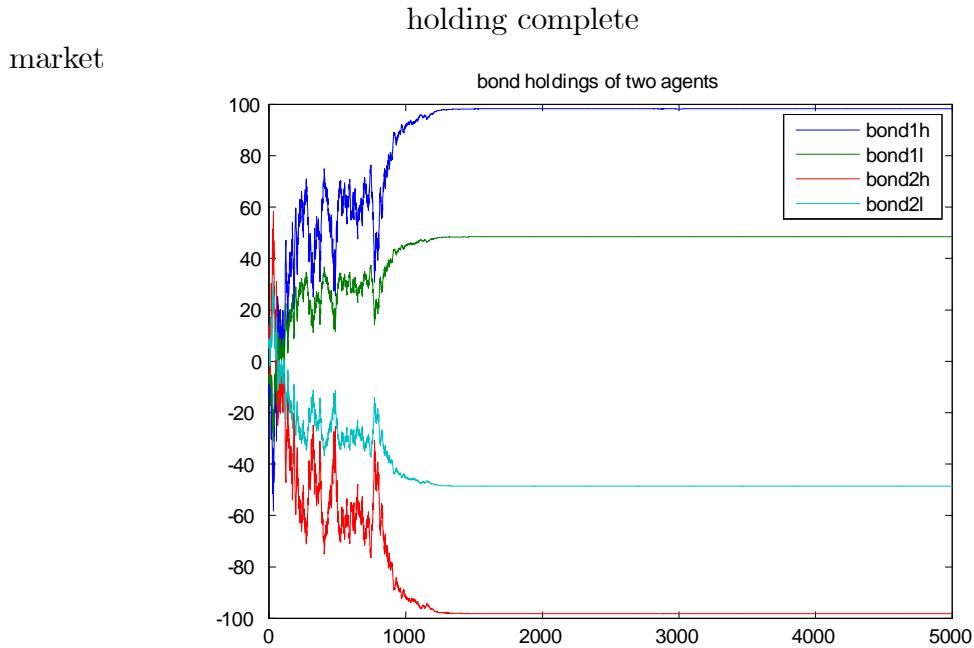
which links SDF of optimistic agent with that of pessimistic agent.

$$\begin{aligned}
P_t^{o,A} &= E_t^o \left[ \sum_{j=1}^{\infty} (\delta^o)^j \prod_{k=1}^j \frac{C_{t+k-1}^o}{C_{t+k}^o} D_{t+j} \right] \\
&= E_t^o \left[ \sum_{j=1}^{\infty} (\delta^o)^j \prod_{k=1}^j \frac{C_{t+k-1}^p}{C_{t+k}^p} A_{t+j} D_{t+j} \right] \\
&= \sum_{j=1}^{\infty} [v(\delta^p)^j \prod_{k=1}^j \frac{C_{t+k-1}^p}{C_{t+k}^p} D_h + (1 - v(\delta^p)^j) \prod_{k=1}^j \frac{C_{t+k-1}^p}{C_{t+k}^p} D_l] \\
&= E_t^p \left[ \sum_{j=1}^{\infty} (\delta^p)^j \prod_{k=1}^j \frac{C_{t+k-1}^p}{C_{t+k}^p} D_{t+j} \right] \\
&= P_t^{p,A}
\end{aligned}$$

Above equation shows that there is an unique pricing SDF in the complete market, hence both optimistic and pessimistic are marginal investors. Due to the fact that A-share and H-share release the same amount of dividend at each period, there is no price difference in each period i.e  $P_t^{o,A} = P_t^{p,A} = P_t^{o,H} = P_t^{p,H}$ . Although the SDFs of the two agents are different and linked by  $A_{t+1}$ , the subjectively expected SDFs are still the same. Hence the diverse beliefs on dividends can't give rise to any price difference.

In addition to analytical solution we can also solve this model by approximating expectation in Euler equations with exponentiated log linear polynomial, and find that agents achieve full insurance through contingent bond exchange. The algorithm for this case is on appendix 2.8.2. Agent 1 is more right with respect to the true probability, and so he accumulates assets and consume more while agent 2 accumulates debt and consume less. In the long run agent 1 consumes the total dividend while agent 2 get nothing. This is consistent with Bloom and Easley (2006).

The bond holding converges to steady state value, which is 100 in our setting. The bond holdings are shown on figure 3. However, even if the two agents have diverse beliefs about the economy situation, price difference still remains in silence. Hence the diverse belief for the dividend or economy only leads the two agents to hold opposite amount of bonds but not to regard the stock price differently. We also find that it is the relative rightness of the perceived beliefs that drive the more right agent accumulate bonds and the other agent accumulate debt rather than their degree of optimism. For



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Figure 3: Agent's holding of contingent bonds

example, even if agent 1 is relatively optimistic, he will also hold bond because of his information advantage. We also try different degree of risk aversion in order to maintain consistency with rational expectation cases. We find that the results of no price difference remain true, and the bond holdings in the long run converge to another steady state level. If we add dividend tax in this context, the price difference occurs but are not quantitatively desirable in the same way that the rational expectation case does.

### 3.3 Models in Incomplete Market

#### 3.3.1 Rational Expectation

Through out the previous section, the assumption of complete market plays a key role in producing one unique SDF which enables us to derive analytical price ratio formula. Under the circumstance of incomplete market, agents can't trade Arrow securities freely to adjust SDF, which seems to be a problem at the first glance. So we suspect that incomplete market perhaps

make some difference. Hence in the current section, we turn to investigate on incomplete market.

We then consider an environment without state contingent bond. The simple discrete dividend process is no longer appropriate for incomplete market. We follows the literature and assume a standard dividend process as

$$\frac{D_t}{D_{t-1}} = a\epsilon_t^d$$

where  $\log\epsilon_t^d \sim i.i.N(-\frac{s_d^2}{2}, s_d^2)$  and  $a \geq 1$ . And budget constraint in this case for  $i = 1, 2$  becomes

$$S_t^{i,A} P_t^A + S_t^{i,H} P_t^H + C_t^i = S_{t-1}^{i,A} (P_t^A + (1 - \tau^{i,A}) D_t^A) + S_{t-1}^{i,H} (P_t^H + (1 - \tau^{i,H}) D_t^H)$$

To avoid Ponzi scheme, the standard no-short-selling constraint is assumed

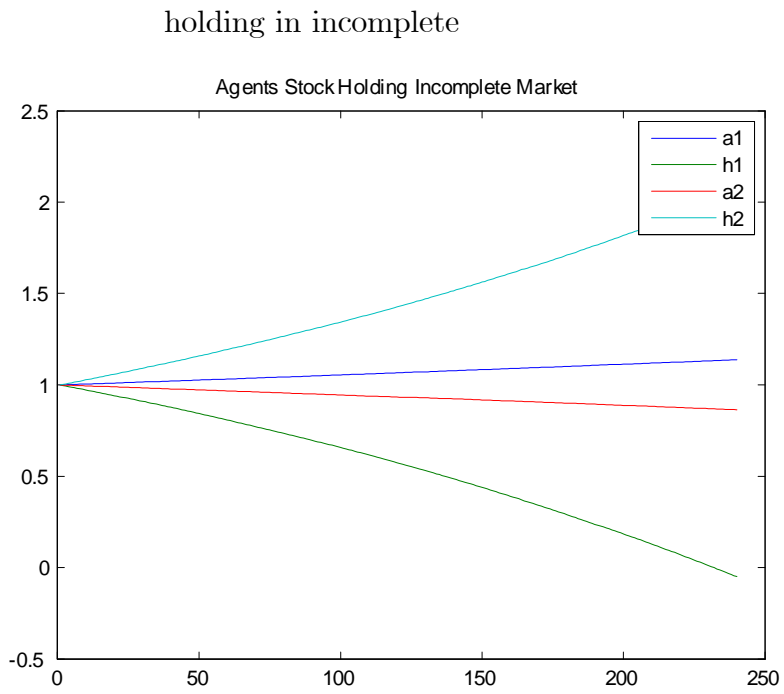
$$S_t^{i,j} \geq 0, \forall i = 1, 2 \forall j = A, H$$

Typically we don't obtain analytical solutions for price ratio when it comes to incomplete market because we are not equipped with the equation that links the two agents' SDFs. Here to keep parsimonious we assume that two types of agents have the same risk aversions and discount factors, but have different dividend taxes. In the later, we will argue that different risk aversions and discount factors cannot generate any AH premium in the incomplete market. However dividend taxes have to be kept in this economy.

We solve the model numerically using Parameterized Expectation Algorithm (PEA) as Marcet and Singleton (1999). We find that during the dynamics, agent 1 holds more and more A shares and less and less H shares while agent 2 does the opposite. This is attributed to the dividend tax structure with  $\tau^{1,A} < \tau^{2,A}$  and  $\tau^{2,H} < \tau^{1,H}$ . However  $P_t^A$  and  $P_t^H$  are the same during a long period because the four F.O.Cs all hold with equality and the lower bound is reached only after a long time. Hence the two agents are both marginal agent for a long time.

This observation is illustrated in figure ???. Once the asset holding reaches to the lower bound, as is in the complete market the price difference quantitatively deficient occurs again as a result of dividend tax structure and this case is degenerated to a 'autarky' world in the sense that mainland investors only hold A-share and foreign investors H-share.

market



4.pdf

Figure 4: Agents' Bond Holding in Incomplete Market

### 3.3.2 Diverse Belief

As in the complete market, we also turn to the case with diverse beliefs. Then agents may have subjective beliefs on dividend process.

$$\frac{D_t}{D_{t-1}} = a_i \epsilon_t^d, \forall i = 1, 2$$

where larger  $a_i$  is associated with optimism and smaller  $a_i$  with pessimism. We solve the two agents two assets incomplete market model with PEA method. We find that the result in 2.3.2 remains. Hence the diverse belief has nothing to do with price difference even in the incomplete market.

## 4 A General Discussion on Sources of Price Difference under Present-Value Model

We of course have not covered every possible model and it is also impossible to do it. However we give a general discussion here on the criteria determining whether a given factor has the potential to drive AH premium in the connected markets. In this section we will discuss the necessary conditions to generate price difference in a general way nesting all the cases we have showed in the above analysis and other cases we have not covered yet.

### 4.1 Variations across Agents

**Proposition 1: Variations across agents has nothing to do with price difference**

This proposition means that prices of the two shares with same dividend stream are same in each period if there are only variations across agents but not variations across two shares. If type  $i$  agent is marginal investor in A-share market, then from F.O.C we have

$$P_t^A = E_t^i f(SDF_t^i, P_{t+1}^A, D_{t+1}, z_{i,A})$$

where  $f$  is a generic function and  $z_{i,A}$  represents all possible market specific factors such as transaction cost and market liquidity.  $E_t^i$  can capture agent  $i$ 's expectation on fundamentals.  $SDF^i$  could be of any type such as the habit type in Campbell and Cochrane (1999) and the long run risk type in



Bansal and Yaron (2004). Functional form of  $f$  should be the same for a given agent across shares in equilibrium. Furthermore the functional form could vary with different models so we denote as abstract function  $f$ . Therefore  $f$  accommodates to any model with variation across agents that we have mentioned or those have not come up with yet.

When we only have variations across agents, then  $z_{1,A} = z_{1,H} = z_1$  and  $z_{2,A} = z_{2,H} = z_2$  even though  $z_1 \neq z_2$ . Then agent  $i$  is also the marginal investor in H-share market. The above F.O.C also applies to H-share. That is

$$P_t^H = E_t^i f(SDF_t^i, P_{t+1}^H, D_{t+1}, z_{i,H})$$

Then A-share and H-share entertain exact the same F.O.Cs in this case. We use  $m_t$  to denote the marginal investor pricing the asset in period  $t$  in equilibrium:

$$m_t = \arg \max_{i \in \{1,2\}} E_t^i f(SDF_t^i, P_{t+1}, D_{t+1}, z_i)$$

By mapping from fundamentals to stock price and assuming no bubbles we obtain

$$P_t^A = E_t^{m_t} g(\{SDF_{t+j-1}^{m_{t+j-1}}\}_{j=1}^{\infty}, \{D_{t+j}\}_{j=1}^{\infty}, \{z_{m_{t+j-1}}\}_{j=1}^{\infty})$$

$$P_t^H = E_t^{m_t} g(\{SDF_{t+j-1}^{m_{t+j-1}}\}_{j=1}^{\infty}, \{D_{t+j}\}_{j=1}^{\infty}, \{z_{m_{t+j-1}}\}_{j=1}^{\infty})$$

Hence prices for A-share and H-share are the same for each period when we only have variations across agents without variations across two shares. Intuitively if there is nothing different across shares, they are the same goods. Then no matter how equilibrium prices are determined in the present-value model, there should not be any price difference. Thus diverse belief, different discount factors and different risk aversions among the two agents do not give rise to the price difference.

## 4.2 Variations across Shares leading to Price Difference

We have to make a given agent regard the A-share and H-share differently even though the same dividend rather than make something different across the two agents, since connection enables two types agents to join into one trading group. Transaction cost is one of the variations across shares. Generally, the transaction cost here includes financial tax, cost of changing currency

and exchange rate change. The financial tax in Hong Kong stock market is 0.118%, in Shanghai is 0.169%. Currency change cost is less than 0.5% through Shanghai-Hong Kong Connect Program. And Hong Kong Dollar is expected to appreciate at mean 1.64% against RMB measured by exchange rate future. So such small transaction cost is impossible to produce desirable quantitative results. Government control can be another variation. Some people hold the long-standing view that Chinese government directly control Shanghai stock market frequently. But this is not true. Since 2000 it only happens one time that when Shanghai stock price bubble burst at the end of June 2015, Chinese government required state-owned investment banks to support stock price by taking long positions to avoid severe financial crisis in the worry of the high leverage held by many Chinese investors. When stock prices was stabilized in August, Chinese governments intervention quickly stepped away.

Another point is about liquidity. The higher liquidity of the stock, the higher price of it. One popular measure of the liquidity is the proportion of no-price-change days of a stock over the sample period (Mei, Scheinkman and Xiong, 2009). Based on daily data for the period 2006-2016, A-share averaged 0.65% of trading days with no price changes, while the corresponding H-share averaged 1.05%. This suggests that A-share is just a little bit more liquid than H-share. We doubt the small difference of liquidities can produce such high and volatile AH premium.

We want to provide a parsimonious way to understand AH premium. When agents don't know the pricing mapping from fundamentals to stock prices and behave like speculators, agents can have different beliefs about capital gains between A-share and H-share markets. Agent's different beliefs could make the agent think A-share and H-share not the same stock, which matches bankers, traders and normal Chinese people's view on the stock market. We are not claiming that we know exactly what are going on in their mind but this sort of story is a dominant view in Chinese market. Hence in the following section we turn to a simple learning model.

## **5 An "Internal Rationality" Learning Model**

Section 2.3 and 2.4 have shown that heterogeneous agents present-value asset pricing models in either complete or incomplete markets are not able to generate sufficient AH premium. This section, hence, presents an "Internal

Rationality" learning model based on Adam, Marcet and Nicolini (2016) to explain such high and volatile AH premium.

## 5.1 Model Environment

A unit of AH-share stock with dividend claim  $D_t$  can be traded in both Shanghai and Hong Kong markets. In addition to  $D_t$ , each agent receives an endowment  $Y_t$  of perishable consumption goods. So the total supply of the consumption goods in the economy is then given by the feasibility condition  $C_t = Y_t + 2D_t$ . Following traditional setting in asset pricing literature, dividend and endowment growth rates follow i.i.d. lognormal processes

$$\frac{D_t}{D_{t-1}} = a\epsilon_t^d, \log \epsilon_t^d \sim iiN\left(-\frac{s_d^2}{2}, s_d^2\right)$$

$$\frac{C_t}{C_{t-1}} = a\epsilon_t^c, \log \epsilon_t^c \sim iiN\left(-\frac{s_c^2}{2}, s_c^2\right)$$

where endowment and dividend growth rates share the same mean  $a$ , and  $(\log \epsilon_t^d, \log \epsilon_t^c)$  are joint-normally distributed with correlation  $\rho_{y,d}$ , and  $s_d$  and  $s_c$  are standard deviations of this joint normal distribution.

The economy is populated by a unit mass of infinite-horizon agents. We model each agent  $i \in [0, 1]$  to have the same standard time-separable CRRA utility function and the same subjective beliefs. This fact, however, is not the common knowledge among agents.

The specification of agent  $i$ 's expected life-time utility function is

$$E^{\mathcal{P}} \sum_{t=0}^{\infty} \delta^t \frac{(C_t^i)^{1-\gamma}}{1-\gamma}$$

where  $C_t^i$  is the consumption profile of agent  $i$ ,  $\delta$  denotes the time discount factor, and  $\gamma$  is the risk-aversion parameter. Instead of the objective probability measure, expectation is formed using the subjective probability measure  $\mathcal{P}$  that describes probability distributions for all external variables. Section 2.5.2 contains more details.

Agent's choices are subjected to standard budget constraint as following

$$C_t^i + P_t^A S_t^{A,i} + P_t^H S_t^{H,i} = (P_t^A + D_t) S_{t-1}^{A,i} + (P_t^H + D_t) S_{t-1}^{H,i} + Y_t$$

where  $S_t^{A,i}$ ,  $S_t^{H,i}$ ,  $P_t^A$  and  $P_t^H$  are defined as section 2.3. To avoid Ponzi schemes and to insure existence of a maximum the following bounds are assumed to hold

$$\begin{aligned}\underline{S} &\leq S_t^{A,i} \leq \bar{S} \\ \underline{S} &\leq S_t^{H,i} \leq \bar{S}\end{aligned}$$

We only assume the bounds  $\underline{S}$  and  $\bar{S}$  are finite.

## 5.2 Probability Space

This subsection explicitly describes the general joint probability space of the external variables. In the competitive economy, each agent considers the joint process of endowment, dividend and stock prices  $\{Y_t, D_t, P_t^A, P_t^H\}$  as exogenous to his decision problem. Rational expectations imply that agents exactly know the mapping from a history of endowment  $Y_t$  and dividend  $D_t$  to equilibrium stock price  $P_t^A$  and  $P_t^H$ . Stock prices hence just carry redundant information. But if the rational expectation assumption is relaxed, as shown in Adam and Marcet (2011) such that agents do not know such mapping because of the non-existence of common knowledge on agents' identical preferences and beliefs, then equilibrium stock price  $P_t^A$  and  $P_t^H$  should be included in the underlying state space. We then define the probability space as  $(\mathcal{P}, \mathcal{B}, \Omega)$  with denoting the corresponding  $\sigma$ -Algebra of Borel subsets of  $\Omega$  and  $\mathcal{P}$  denoting the agent's subjective probability measure over  $(\mathcal{B}, \Omega)$ . The state space  $\Omega$  of realized exogenous variables is

$$\Omega = \Omega_Y \times \Omega_D \times \Omega_{P^A} \times \Omega_{P^H}$$

where  $\Omega_X$  is the space of all possible infinite sequences for the variable  $X \in [Y, D, P^A, P^H]$ . Thereby, a specific element in the set  $\Omega$  is an infinite sequence  $\omega = \{Y_t, D_t, P_t^A, P_t^H\}_{t=0}^\infty$ . The expected utility with probability measure  $\mathcal{P}$  is defined as

$$E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \delta^t \frac{(C_t^i)^{1-\gamma}}{1-\gamma} \equiv \int_{\Omega} \sum_{t=0}^{\infty} \delta^t \frac{C_t^i(\omega)^{1-\gamma}}{1-\gamma} d(\omega)$$

Agent  $i$  makes contingent plans for endogenous variables  $C_t^i$ ,  $S_t^{A,i}$ ,  $S_t^{H,i}$  according to the policy function mapping in the following

$$(C_t^i, S_t^{A,i}, S_t^{H,i}) : \Omega^t \rightarrow R^3$$

where  $\Omega^t$  represents the set of histories from period zero up to period  $t$ .

### 5.3 Optimality Conditions

Since the objective function is concave and the feasible set is convex, the agent's optimal plan is characterized by the first order conditions

$$(C_t^i)^{-\gamma} P_t^A = \delta E_t^{\mathcal{P}}((C_{t+1}^i)^{-\gamma}(P_{t+1}^A + D_{t+1})) \quad (5)$$

$$(C_t^i)^{-\gamma} P_t^H = \delta E_t^{\mathcal{P}}((C_{t+1}^i)^{-\gamma}(P_{t+1}^H + D_{t+1})) \quad (6)$$

Before exploring why subjective beliefs can explain AH premium, we first briefly review the unique RE solution given by

$$P_t^{A.RE} = \frac{\delta a^{1-\gamma} \rho_\epsilon}{1 - \delta a^{1-\gamma} \rho_\epsilon} D_t \quad (7)$$

$$P_t^{H.RE} = \frac{\delta a^{1-\gamma} \rho_\epsilon}{1 - \delta a^{1-\gamma} \rho_\epsilon} D_t \quad (8)$$

where  $\rho_\epsilon = E[(\epsilon_{t+1}^c)^{1-\gamma} \epsilon_{t+1}^d] = e^{\gamma(1+\gamma)\frac{s_c^2}{2}} e^{-\gamma\rho_{c,d} s_c s_d}$ . Obviously, RE solutions always generate  $P_t^{A.RE} = P_t^{H.RE}$ .

We now characterize the equilibrium outcome under learning. According to the arguments in Adam, Marcet and Nicolini (2016), out of strict rational expectations we may have  $E[C_{t+1}^i] \neq E_t[C_{t+1}^i]$  even if in the equilibrium  $C_{t+1}^i = C_{t+1}^i$  holds ex-post. But we can make the same approximations in the following as they do

$$E_t^{\mathcal{P}}[(\frac{C_{t+1}^i}{C_t^i})^{-\gamma}(P_{t+1}^A + D_{t+1})] \simeq E_t^{\mathcal{P}}[(\frac{C_{t+1}^i}{C_t^i})^{-\gamma}(P_{t+1}^A + D_{t+1})] \quad (9)$$

$$E_t^{\mathcal{P}}[(\frac{C_{t+1}^i}{C_t^i})^{-\gamma}(P_{t+1}^H + D_{t+1})] \simeq E_t^{\mathcal{P}}[(\frac{C_{t+1}^i}{C_t^i})^{-\gamma}(P_{t+1}^H + D_{t+1})] \quad (10)$$

The following assumption provides sufficient conditions for this to be the case:

**Assumption 1** We assume that  $Y_t$  is sufficiently large and the  $E_t^{\mathcal{P}} P_{t+1}^{A(H)} / D_t < \bar{M}$  for some  $\bar{M} < \infty$  so that, given finite asset bounds  $\underline{S}$  and  $\bar{S}$ , the approximations (9) and (10) hold with sufficient accuracy.

We then can define the subjective expectations of risk-adjusted stock price growths as

$$\beta_t^A \equiv E_t^{\mathcal{P}}[(\frac{C_{t+1}^i}{C_t^i})^{-\gamma}(P_{t+1}^A)] \quad (11)$$

$$\beta_t^H \equiv E_t^{\mathcal{P}}\left[\left(\frac{C_{t+1}^i}{C_t^i}\right)^{-\gamma}(P_{t+1}^H)\right] \quad (12)$$

We also assume that agents know the true processes of consumption and dividend growths. The definitions of  $\beta_t^A$  and  $\beta_t^H$  together with two first order conditions (5) and (6) give rise to the asset pricing equations

$$P_t^A = \frac{\delta a^{1-\gamma} \rho_\epsilon}{1 - \delta \beta_t^A} D_t \quad (13)$$

$$P_t^H = \frac{\delta a^{1-\gamma} \rho_\epsilon}{1 - \delta \beta_t^H} D_t \quad (14)$$

From equation (13) and (14), our learning model is possible to generate AH premium if  $\beta_t^A \neq \beta_t^H$ .

## 5.4 Beliefs Updating Rule

This section fully specifies the subjective probability distribution  $\mathcal{P}$ , and derives the optimal belief updating rule for subjective beliefs  $\beta_t^A$  and  $\beta_t^H$ . Similar to the arguments in Adam, Marcet and Nicolini (2016), in agents' beliefs the processes for risk-adjusted stock price growths in both Shanghai and Hong Kong markets can be modeled as the sum of a persistent component and of a transitory component

$$\begin{aligned} \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{P_{t+1}^A}{P_t^A} &= e_t^A + \epsilon_t^A, \quad \epsilon_t^A \sim iiN(0, \sigma_{\epsilon,A}^2) \\ e_t^A &= e_{t-1}^A + \xi_t^A, \quad \xi_t^A \sim iiN(0, \sigma_{\xi,A}^2) \end{aligned} \quad (15)$$

$$\begin{aligned} \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{P_{t+1}^H}{P_t^H} &= e_t^H + \epsilon_t^H, \quad \epsilon_t^H \sim iiN(0, \sigma_{\epsilon,A}^2) \\ e_t^H &= e_{t-1}^H + \xi_t^H, \quad \xi_t^H \sim iiN(0, \sigma_{\xi,A}^2) \end{aligned} \quad (16)$$

where  $e_t^A$  and  $e_t^H$  are persistent components,  $\epsilon_t^A$  and  $\epsilon_t^H$  are transitory components. One way to justify these processes is that they are compatible with RE. According to equation (7) and (8), the rational expectation of risk-adjusted price growth is  $E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{P_{t+1}^A}{P_t^A}\right] = E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{P_{t+1}^H}{P_t^H}\right] = a^{1-\gamma} \rho_\epsilon$ .

Hence, the previous setup encompasses the rational expectation equilibrium as a special case when agents believe  $\sigma_{\xi,A}^2 = \sigma_{\xi,H}^2 = 0$  and assign probability one to  $e_0^A = e_0^H = a^{1-\gamma}\rho_\epsilon$ .

Then, we allow for a non-zero variance  $\sigma_{\xi,A}^2$  and  $\sigma_{\xi,H}^2$ . Agents can only observe the realizations of risk-adjusted growths (the sum of persistent and transitory components), hence the requirement to forecast the persistent components  $e_t^A$  and  $e_t^H$  calls for a filtering problem. The priors of agents' beliefs can be centered at some initial values and given by

$$e_0^A \sim N(\beta_0^A, \sigma_{0,A}^2)$$

$$e_0^H \sim N(\beta_0^H, \sigma_{0,H}^2)$$

and the variances of prior distributions should be set up to equal with steady state Kalman filter uncertainty about  $e_t^A$  and  $e_t^H$

$$\sigma_{0,A}^2 = \frac{-\sigma_{\xi,A}^2 + \sqrt{\sigma_{\xi,A}^4 + 4\sigma_{\xi,A}^2\sigma_{\epsilon,A}^2}}{2}$$

$$\sigma_{0,H}^2 = \frac{-\sigma_{\xi,H}^2 + \sqrt{\sigma_{\xi,H}^4 + 4\sigma_{\xi,H}^2\sigma_{\epsilon,H}^2}}{2}$$

Then agents' posterior beliefs will be

$$e_t^A \sim N(\beta_t^A, \sigma_{0,A}^2)$$

$$e_t^H \sim N(\beta_t^H, \sigma_{0,H}^2)$$

And the optimal updating rule implies that the evolution of  $\beta_t^A$  and  $\beta_t^H$  is taking the form just as constant gain adaptive learning

$$\beta_t^A = \beta_{t-1}^A + \frac{1}{\alpha^A} \left( \left( \frac{C_{t-1}}{C_{t-2}} \right)^{-\gamma} \frac{P_{t-1}^A}{P_{t-2}^A} - \beta_{t-1}^A \right) \quad (17)$$

$$\beta_t^H = \beta_{t-1}^H + \frac{1}{\alpha^H} \left( \left( \frac{C_{t-1}}{C_{t-2}} \right)^{-\gamma} \frac{P_{t-1}^H}{P_{t-2}^H} - \beta_{t-1}^H \right) \quad (18)$$

where  $\alpha^A = \frac{\sigma_{\xi,A}^2 + \sqrt{\sigma_{\xi,A}^4 + 4\sigma_{\xi,A}^2\sigma_{\epsilon,A}^2}}{2\sigma_{\xi,A}^2}$  and  $\alpha^H = \frac{\sigma_{\xi,H}^2 + \sqrt{\sigma_{\xi,H}^4 + 4\sigma_{\xi,H}^2\sigma_{\epsilon,H}^2}}{2\sigma_{\xi,H}^2}$  given by optimal (Kalman) gain.

The adaptive learning schemes as equation (17) and (18) as well as pricing equation (13) and (14) can generate a high stock markets volatility coming from the feedback channel between stock price  $P_t^{A(H)}$  and subjective beliefs  $\beta_t^{A(H)}$ . According to equation (13) or (14), a high (low)  $\beta_t^{A(H)}$  will lead to a high (low) realized stock price. This will reinforce the subjective beliefs to induce a even higher (lower)  $\beta_{t+1}^{A(H)}$  through equation (17) or (18) leading to much higher (lower) stock price so on. The self-referential aspect of the model is the key for producing stock market dynamics. Therefore, a difference of initial beliefs between  $\beta^A$  and  $\beta^H$  or of learning speeds  $\alpha^A$  and  $\alpha^H$  is promising to generate persistently different prices between A-share and H-share.

Finally, in order to avoid the explosion of stock price  $P_t^{A(H)}$  agents' subjective belief  $\beta_t^{A(H)}$  is replaced by  $\omega(\beta_t^{A(H)})$ , the projection facilities as appendix 2.8.4.

## 5.5 Testing for the Rationality of Price Expectation

In this section we use a set of testable restrictions implied by agents' beliefs system developed in Adam, Marcet and Nicolini (2016). These restrictions are listed as follows: Denote  $x_t = (e_t, D_t/D_{t-1}, C_t/C_{t-1})$ , where  $e_t \equiv \Delta(\frac{C_t}{C_{t-1}})^{-\gamma} \frac{P_t}{P_{t-1}}$  with  $\Delta$  denoting the first difference operator.

Restriction 1:  $E(x_{t-i}e_t) = 0$  for all  $i \geq 2$ ,

Restriction 2:  $E((\frac{D_t}{D_{t-1}} + \frac{D_{t-1}}{D_{t-2}}, \frac{C_t}{C_{t-1}} + \frac{C_{t-1}}{C_{t-2}})e_t) = 0$ ,

Restriction 3:  $b'_{DC} \sum_{DC} b_{DC} + E(e_t e_{t-1}) < 0$ ,

Restriction 4:  $E(e_t) = 0$ ,

where  $\sum_{DC} \equiv var(\frac{D_t}{D_{t-1}}, \frac{C_t}{C_{t-1}})$  and  $b_{DC} \equiv \sum_{DC}^{-1} E((\frac{D_t}{D_{t-1}}, \frac{C_t}{C_{t-1}})' e_t)$

These four restrictions are necessary and sufficient conditions for the agents' belief compatible with  $\{x_t\}$  in terms of second order moments. Adam, Marcet and Nicolini (2016) proves that under standard assumptions, any process satisfying these testable restrictions can - in terms of its autocovariance function - be generated by the postulated system of beliefs as (15) and (16). The set of derived restrictions thus fully characterizes the second-moment implications of the beliefs system. Here we test the derived restrictions against the data to see if the agent's belief system is compatible with the actual data. Table 3 reports the test statistics when testing Restrictions 1-4 using actual data. We compute risk-adjusted consumption growth in the data at  $\gamma = 5$ .



	Test Statistics A (H)	5% Critical Value
Restriction 1 using $\frac{D_t}{D_{t-i-1}}$	2.81 (0.76)	9.48
Restriction 1 using $\frac{C_t}{C_{t-i-1}}$	4.02 (4.77)	9.48
Restriction 1 using $\Delta\left(\frac{C_{t-i}}{C_{t-i-1}}\right)^{-\gamma} \frac{P_{t-i}}{P_{t-i-1}}$	2.13 (2.55)	9.48
Restriction 2	0.04 (0.15)	5.99
Restriction 3	-3.55 (-3.60)	3.84
Restriction 4	0.002 (0.001)	1.64

Table 3: Testing Subjective Beliefs against Actual Data

The 5% critical value of the test statistic is reported in the last column of table **3**. The table shows that the test statistic is in all cases below its critical value and often so by a wide margin. It then follows that agents find the observed asset pricing data, in terms of second moments, to be compatible with their belief system. Based on this, we can conclude that the agents' belief system is reasonable: given the behavior of actual data, the belief system is one that agents could have entertained.

## 5.6 Quantitative Performance

This subsection presents the simulation outcomes of our learning model. We simulate our model at weekly frequency. We first give value to the coefficient of relative risk-aversion  $\gamma$  at 5 following Adam, Marcet and Nicolini (2016), then calibrate the mean and standard deviation of dividend growth  $a$ ,  $\sigma_{\Delta D/D}$ , the standard deviation of consumption growth  $\sigma_{\Delta C/C}$ , the correlation between consumption growth and dividend growth  $\rho_{c,d}$  using Shanghai stock market data and Chinese consumption per capita data. We also calibrate  $\delta$  to match annual 4% interest rate. Meanwhile, we give values to  $\alpha^A$  and  $\alpha^H$  such that  $\alpha^A < \alpha^H$ , which can come from agents' subjective beliefs that  $\frac{\sigma_{\xi,A}}{\sigma_{\epsilon,A}} > \frac{\sigma_{\xi,H}}{\sigma_{\epsilon,H}}$ . Intuitively, if agents believe that the ratio of standard deviation of persistent component shock to that of transitory component is relative larger in A-share price than H-share price, agents prefer to learn faster for A-share price since only persistent component provides useful information for forecasting. This is not arbitrary setting because the realized data of  $P_t^A$  and  $P_t^H$  can support this inequality if we use MLE method to estimate related parameters given the data following the processes **(15)** and **(16)**. Table **4**

Parameters	Value
$\gamma$	5
$\sigma_{\Delta D/D}$	0.0204
$\sigma_{\Delta C/C}$	0.0025
$a$	1.0014
$\rho_{c,d}$	-0.03
$\delta$	0.999
$1/\alpha^A$	0.0030
$1/\alpha^H$	0.0015

Table 4: Parameters Values for Learning Model

contains the parameter values.

We Monte-Carlo simulate the learning model for  $K = 10,000$  samples with each sample having  $T = 100$  periods to match almost 2 years' sample period since November 2014. Table 5 contains the simulation results. Column 2 shows the data moments of AH premium, and column 3 has the 95% interval of model simulated moments. We find that the mean and standard deviation of data locate in the interval, but model generates a little more persistent price difference than data. And figure 5 also presents one simulated dynamics of A-share price  $P_t^A$  and H-share price  $P_t^H$ , and figure 6 presents the corresponding simulated AH premium. We set initial conditions  $\beta_1^A = \beta_1^H$  and  $\beta_2^A$  a little larger than  $\beta_2^H$ , which are consistent with data observations. Then, a higher learning speed in A-share leads  $P_t^A$  to fluctuate more strongly than  $P_t^H$  even if two prices dynamics keep the similar shape. Comparing with figure, the model simulated prices display the close shape as data. More importantly, the shape of simulated AH premium captures several important factors of data: 1. starting from around 100; 2. persistently increasing to about 150; and 3. decreasing to about 120 after 2 years. Hence, our learning model does a much better job in generating data-like AH premium than the models in section 2.3.

## 6 Convergence Traders' Strategy

A typical convergence trader is to bet that price difference between two assets with identical, or similar fundamentals will narrow in the future. The con-

Moments	Data	Model
$E(\frac{P_t^A}{P_t^H} * 100)$	130.39	[119.58 197.01]
$\sigma(\frac{P_t^A}{P_t^H} * 100)$	9.53	[7.75 48.69]
$\rho(\frac{P_t^A}{P_t^H} * 100)$	0.78	[0.87 0.98]

Table 5: Model Simulated Moments

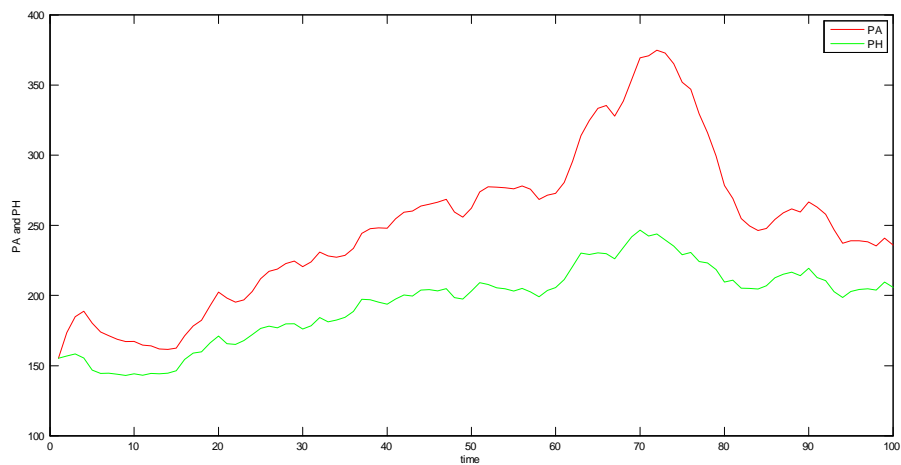


Figure 5: Simulated Stock Prices of A-Share and H-Share

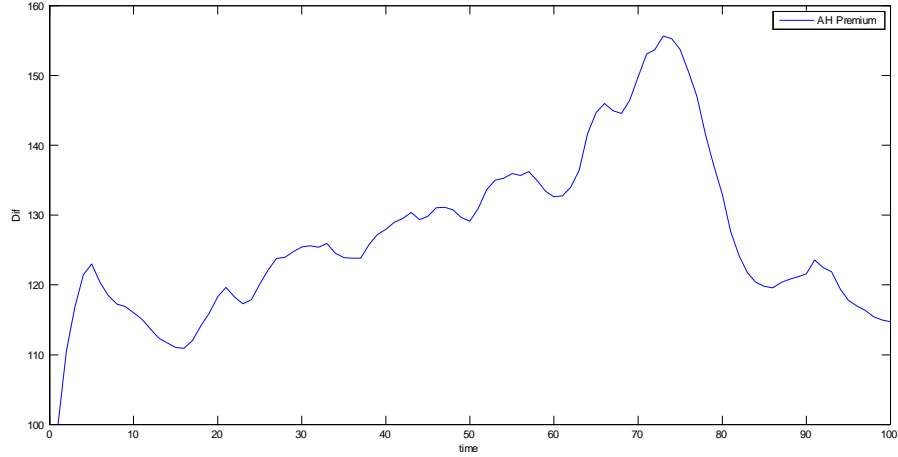


Figure 6: Simulated AH Premium

vergence trade would hold long positions in one asset he considers undervalued and short positions in the other asset he considers overvalued. A famous example is that the hedge fund Long-Term Capital Management (LTCM) expected the convergence of bond yields in the emerging market countries and US (Edwards, 1999). They bought emerging markets' bonds and sold short US government bonds. The spread of bond yields, however, widened because of the deterioration of Asian financial crisis and the default of Russian Sovereign debt. The unexpected widening leads to the near-collapse of LTCM. Besides the case of LTCM, Wall Street Journal reported in June 2015 that many convergence traders participated in AH-share market by short selling A-share and long buying H-share, but finally encountered a huge loss.

Xiong (2001) studies the convergence trading strategy in the model which has three types of traders: noise traders, convergence traders and long-term traders. He finds that convergence traders reduces asset price volatility in general, but when an unfavorable shock causes them to suffer substantial capital losses, they liquidate their positions, thereby amplifying the original shock. This section considers the convergence traders differently with zero measure who take the stock prices set by learning agents as given, and studies the probability distribution of profits when they hold convergence trader strategy.

$m_t$	Mean	Std	$\Pr(m_t < 0)$
3m	0.817	10.657	0.384
6m	0.662	17.345	0.365
9m	0.713	23.231	0.338
1y	0.887	26.659	0.323

Table 6: The Distribution of Profits from Convergence Trading Strategy

The convergence traders at period 100 expect that AH premium should narrow in the future, hence they short sells 1 unit of A-share stock and use the money from selling to buy H-share stock. To implement short selling in China stock market, convergence traders should have as much money as 50% of short selling value in their account as security deposit. In every period, the guarantee ratio  $gr_t$  should be calculated as

$$gr_t = \frac{0.5 * P_{100}^A + P_t^H * \frac{P_{100}^A}{P_{100}^H}}{P_t^A}$$

If  $gr_t < 130\%$ , convergence traders will be asked to add more security deposit to avoid forced liquidation. The maximum duration of short selling is 1 year. We now Monte-Carlo simulate the learning model 10,000 paths with each path representing from 100th period to 152th period. The probability of  $gr_t < 130\%$  in every period can reach as high as 13%. We can also calculate the distribution of profits  $m_t = P_t^H * \frac{P_{100}^A}{P_{100}^H} - P_t^A$  when  $t = 113, 126, 139$  or 152 corresponding to 3 months, 6 months, 9 months and 1 year. Table ?? shows the results. We find the mean of  $m_t$  much smaller than the standard deviation of it and a large probability of losing money. Being different from Xiong (2001), our learning model cannot guarantee the convergence of AH premium. Therefore, it is not surprising that convergence traders have large probability to lose money.

## 7 Conclusion

This chapter studies the AH premium, which is an interesting anomaly in asset markets. We have shown that asset pricing models with heterogeneity agents with different risk aversions or diverse beliefs in the complete market and incomplete markets cannot generate any data-like AH premium. Transaction cost and different dividend taxes between Shanghai and Hong Kong

markets also fails to explain such high and volatile AH premium. We propose an "Internal Rationality" model, in which agents don't know the pricing function from fundamentals to the stock prices and have different subjective beliefs about tomorrow's capital gains in Shanghai and Hong Kong markets. Our learning model can successfully generate data-like weekly AH premium. Finally we show that convergence traders with strategy short in Shanghai and long in Hong Kong will lose money with 33% probability.

This maybe an evidence that Chinese investors are more speculative, which seems to be related to the higher stock price volatility in China than that in U.S. and the fact that stock price is highly negative correlated with PMI index for economy prospect in China during the year 2015. These topics worth to be explored in the future research.

## 8 Appendix

### 8.1 Algorithm for two agents two shares with rational expectation in complete market

Step 1: Simulate N times of  $\{D_t\}$  for T periods. Solve for  $\frac{u'(C_0^1)}{u'(C_0^2)}$  by simulating the economy especially  $\{\{C_t^{1,n}, C_t^{2,n}\}_{t=0}^T\}_{n=1}^N$  given initial bond holding  $B_{-1}$ , since we have one equation of present value budget constraint for  $B_{-1}$  and one unknown. Hence we got the equilibrium  $\lambda$ . It could be solved by iterating on  $\lambda$  or for example just use solve the equation directly. The equation is as follows:

$$\frac{1}{N} \sum_{n=1}^N \sum_{t=0}^T \delta^{1,t} \frac{u'(C_t^1)}{u'(C_0^1)} (C_{t+j}^1 - D_{t+j}) = B_{-1}$$

Step 2: Find  $\{C_t^1, C_t^2\}_{t=0}^T$  by simulating a very long sequence of  $D$ . At time  $t$ , given  $\frac{u'(C_t^1)}{u'(C_t^2)} = \lambda$  and market clear condition,  $C_t^1$  and  $C_t^2$  can be solved. Here we are facing a convex problem. Thus theoretically we should get unique solution though the two conditions lead to a polynomial of  $C$ .

Step 3: Solve for the realized present value of primary deficit  $\{Dd_t^1\}$ . It is useful because the bond holdings are just conditional expectation of  $Dd$

Define  $Dd_t^1 = \sum_{j=0}^{\infty} \delta^{1,j} \frac{u'(C_{t+j}^1)}{u'(C_t^1)} (C_{t+j}^1 - D_{t+j}^1)$  as realized present value of pri-

mary deficit. Then we have

$$Dd_t^1 = \delta^1 \frac{u'(C_{t+1}^1)}{u'(C_t^1)} Dd_{t+1}^1 + C_t^1 - D_t^1.$$

We set  $Dd$  at the end of the day to an arbitrary value, 0 for example, and we can solve for  $Dd$  backwards from  $Dd_T^1 = 0$  given that we have got sequence of consumption and dividends in the above steps.

Step 4 : We solve for  $\{B_{t-1}^1(D)\}$  in this step by using the equation:

$$B_{t-1}^1(D_t) = E(Dd_t | D_t = D_t)$$

where bond holding are just function of state  $D$ . To implement it we use

$$B_{t-1}^1(D_t) = \frac{1}{T} \sum_{t=1}^T Dd_t^1 I(D_t)$$

where  $I(D_t)$  is the indicator function. This could also be regarded as run the regression of  $Dd$  to indicator functions, which is the core idea of PEA.

$$Dd_t^1 = \alpha_0^h I_{D^h}(D_t) + \alpha_0^l I_{D^l}(D_t)$$

Notice that conditional expectation is actually the average over states. However due to the fact that we have an i.i.d world which is definitely ergodic, we just use the average over time to estimate the conditional expectation by the property of ergodicity.

Technically speaking we are not using PEA because we are not iterating on parameters, which is not necessary in our case. We are not relying on the approximation of the right hand side of Euler equations as the typical steps do in PEA thanks to the complete market thing gives us the formula to solve for debt and  $Bd$ .

## 8.2 Algorithm for two agents two shares with diverse belief in complete market

In this case every steps are same except that  $\lambda$  is not constant any more. We will have a sequence  $\{\lambda_t\}$  because of the diverse probability, which follows

$$\alpha_{t-1}(D_t) \lambda_{t-1} = \lambda_t$$

where  $\alpha_{t-1}(D_t) = \frac{prob_{t-1}^2(D_t)}{prob_{t-1}^1(D_t)}$ . Another difference lies in step 4 because in this case bond holding is not only the function of  $D$  but also a function of  $\lambda_{t-1}$ . To take a stand on agent 1,

$$B_{t-1}^1(D_t) = E(Dd_t | D_t = D_t, \lambda_{t-1})$$

So we need to run the regression of  $Dd$  on both  $D$  and  $\lambda_{t-1}$ . Explicitly the best way to write  $Dd$  as the function of the two states are as follows:

$$Dd_t^1 = (\alpha_0^h + \alpha_1^h \lambda_{t-1}) I_{D^h}(D_t) + (\alpha_0^l + \alpha_1^l \lambda_{t-1}) I_{D^l}(D_t)$$

Clearly this regression could be run separately both for high and low.

### 8.3 A Learning Model with Diverse Beliefs and Dividend Taxes

This section extends the benchmark learning model with diverse beliefs and dividend taxes. The dividend and consumption growths still follow the same processes as section 2.5. There are two types of agents, one is relative optimistic about fundamental growth and the other is relative pessimistic. Agent  $i$ 's maximization problem for  $i = o$  or  $p$  is

$$\begin{aligned} \max E_0^{\mathcal{P}} \sum_{t=0}^{\infty} \delta^t \frac{(C_t^i)^{1-\gamma}}{1-\gamma} \\ \text{s.t. } C_t^i + P_t^A S_t^{A,i} + P_t^H S_t^{H,i} &= (P_t^A + (1 - \tau^{i,A}) D_t) S_{t-1}^{A,i} + (P_t^H + (1 - \tau^{i,H}) D_t) S_{t-1}^{H,i} + Y_t \\ 0 &\leq S_t^{A,i} \ \& \ 0 \leq S_t^{H,i} \end{aligned}$$

The subjective belief  $\mathcal{P}$  is the same as  $\mathcal{P}$  in previous section except that agent  $i$  believes fundamental growth at rate of  $a^i$  instead of  $a$ . The first order conditions are

$$(C_t^i)^{-\gamma} P_t^A \geq \delta E((C_{t+1}^i)^{-\gamma} (P_{t+1}^A + (1 - \tau^{i,A}) D_{t+1})) \text{ with equality if } S_t^{A,i} > 0$$

$$(C_t^i)^{-\gamma} P_t^H \geq \delta E((C_{t+1}^i)^{-\gamma} (P_{t+1}^H + (1 - \tau^{i,H}) D_{t+1})) \text{ with equality if } S_t^{H,i} > 0$$

We then can define the subjective expectations of risk-adjusted stock price growth as

$$\beta_t^{i,A} \equiv E_t^{\mathcal{P}} \left[ \left( \frac{C_{t+1}^i}{C_t^i} \right)^{-\gamma} (P_{t+1}^A) \right]$$



Parameters	Value
$a^o$	1.0024
$a^p$	1.0004
$\tau^{o,A}$	0.05
$\tau^{p,A}$	0.10
$\tau^{o,H}$	0.20
$\tau^{p,H}$	0.10

Table 7: Parameters Values for Learning Model

$$\beta_t^{i,H} \equiv E_t^{\mathcal{P}} \left[ \left( \frac{C_{t+1}^i}{C_t^i} \right)^{-\gamma} (P_{t+1}^H) \right]$$

And agent  $i$  updates  $\beta_t^{i,A}$  and  $\beta_t^{i,H}$  according to the same adaptive learning schemes as (17) and (18). The consumption good market clearing condition is

$$C_t = C_t^o + C_t^p = 2Y_t + 2D_t$$

Assumption 1 allows us to have the following approximations

$$E_t^{\mathcal{P}} \left[ \left( \frac{C_{t+1}^i}{C_t^i} \right)^{-\gamma} \right] = E_t^{\mathcal{P}} \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right] \text{ for } i = o, p$$

The pricing equations according to Adam and Marcet (2011) are

$$P_t^A = \max_{i \in O, P} \frac{\delta (a^i)^{1-\gamma} \rho_\epsilon}{1 - \delta \beta_t^{i,A}} (1 - \tau^{i,A}) D_t,$$

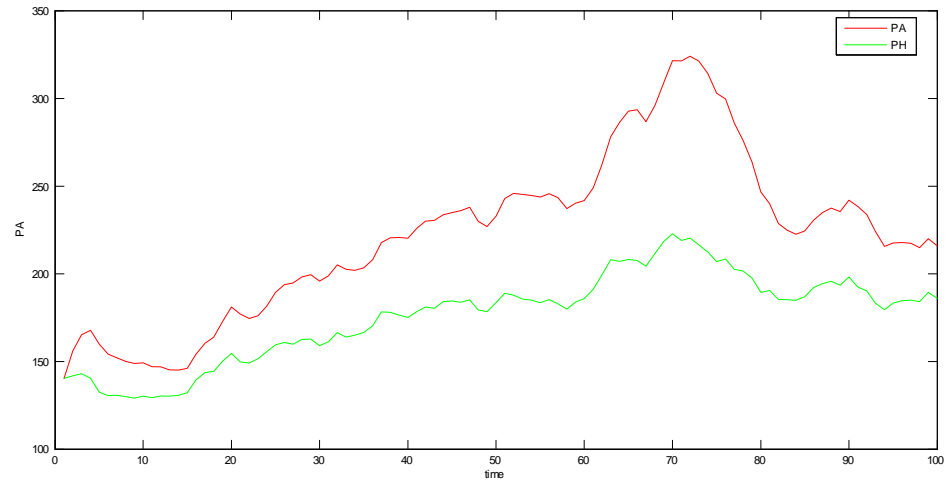
$$P_t^H = \max_{i \in o, p} \frac{\delta (a^i)^{1-\gamma} \rho_\epsilon}{1 - \delta \beta_t^{i,A}} (1 - \tau^{i,A}) D_t,$$

The new parameter values are given in table 7. The simulated stock prices of A-share and H-share is in figure 7, and the simulate AH premium in figure 8. The similar share of AH premium compared with it in figure 6 confirms that different beliefs about capital gains are dominate factor in generating AH premium relative to diverse beliefs on fundamentals and dividend taxes.

## 8.4 Differentiable Projection Facility

The function  $\omega$  used in the differentiable projection facility is

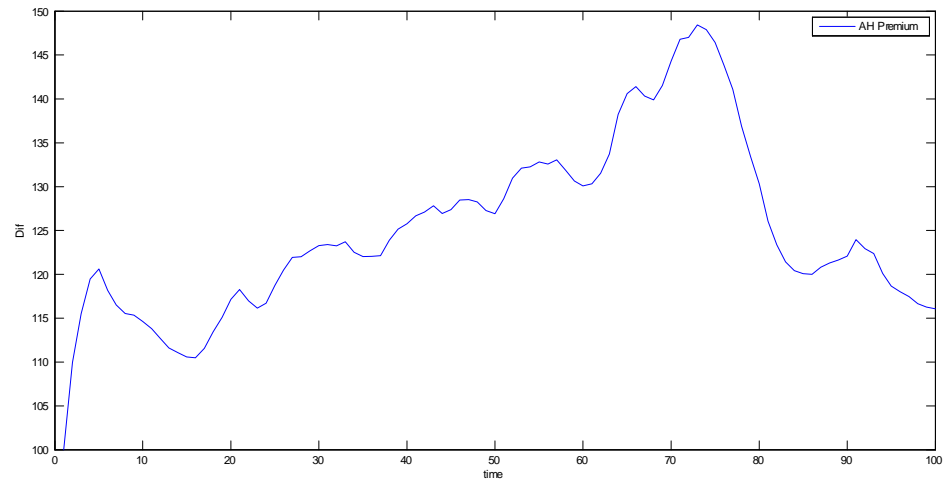
belief



7.pdf

Figure 7: Simulated Stock Prices of A-Share and H-Share

belief



8.pdf

Figure 8: Simulated AH Premium

$$\omega(\beta) = \left\{ \begin{array}{ll} \beta & \text{if } x \leq \beta^L \\ \beta^L + \frac{\beta - \beta^L}{\beta + \beta^U - 2\beta^L}(\beta^U - \beta^L) & \text{if } \beta^L < x \leq \beta^U \end{array} \right\}$$

In our numerical applications, we choose  $\beta^U$  so that the implied PD ratio never exceeds  $U^{PD} = 600$  and  $\beta^L = \delta^{-1} - 2(\delta^{-1} - \beta^U)$ .

## 8.5 Data Sources

Our data set for China stock market price, dividend, Heng Seng China AH premium index, Heng Seng China A index and Heng Seng China H index are downloaded from Wind Financial Database (<http://www.wind.com.cn>). The daily price series has been transformed into a weekly series by taking the index value of the last day of the considered week.

Our data set for Chinese macro data like consumption and CPI are downloaded from Chung, Chen, Waggoner and Zha (2015) (<http://www.tzha.net/code>).

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